ECONOMIC ORDER QUANTITY WITH IMPERFECT QUALITY ITEMS WHERE SHORTAGES ARE BACKLOGGED IN INTUITIONISTIC FUZZY ENVIRONMENT

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Abstract: **This paper discusses inventory situation where all the items are not of perfect quality. They are picked up during the screening process and are shipped in one lot due to economics of sale. Transportation cost is included. Shortages are backordered. Holding cost, shortages cost, Inspection cost, transportation costs are taken as intuitionist triangular fuzzy numbers. Signed distance method is used for defuzzification. An illustrate example is provided in order to obtained optimal order quantity. The sensitivity analysis has been performed to study the effects of change of various charges in total charge.**

Keywords: **Inventory, Intuitionistic fuzzy number, backorder, triangular intuitionistic fuzzy numbers.**

Introduction

Business environment being very dynamic today, survival and growth have become the buzz words. We are always shocked and disturbed when we hear about work place disasters. These reactions primarily focus on the workers who killed or injured. But admidst this expression of humanity questions begins to arise. It should be no surprise that consistent regulatory inspections improve employee safety. Inspections should be an essential parts of any health and safety program. Due to the uncertainty in demand estimations the organizations face a problem of stock outs. Inventory backordered is the situation in which a customer's order cannot be filled when presented, and for which the customer is prepared to wait for some time. The transportation cost is calculated as the expense involved in moving products to a different place. Which are often passed on the customers, a business would generally incur a transportation cost, and it needs to bring its products to retailers in order to have them offered for sale to customers. Suppose the lack of transportation leads to a shortage. The economic order quantity (EOQ) model of (Harris 1913) is the foundation of modern day inventory models. Fuzzy set theory was introduced by Zadeh [16] in 1965. The application of fuzzy set theory to inventory problems has been proposed by Park in 1987 park used fuzzy set concepts to treat the inventory problem with fuzzy inventory cost under arithmetic operations of extension principle. Chen and Hsieh discussed a fuzzy backorder inventory model. Purnomo et al[15] has proposed an EOQ model with partial backorder using median rule. As an extension of the fuzzy set, the concept of intuitionistic fuzzy set (IFS) was introduced by Attanassov [4]. It is characterized by two functions expressing the degree of membership and the degree of non-membership, respectively. An IFS is a very suitable tool to describe the uncertain decision information and deal with the uncertainty and vagueness in decision making. Armand Baboli, Mohammadali Pirayesh Neghab, Rasoul Haji [3] derived an algorithm for the determination of the Economic order quantity in a two level supply chain with Transportation costs: Comparison of decentralized with centralized decision. Banerjee and Roy[5] generalized the application of the in intuitionistic fuzzy optimization in the constrained multi objective stochastic inventory model. Nagoorgani [13] also used triangular intuitionistic fuzzy number in solving a linear programming problem. Chakraborty et al[10] gave the solution for the basic EOQ model using intuitionistic fuzzy optimization technique where in the various parameters including shortage cost are first treated as fuzzy numbers. Mahapatra[12] gave a multi objective inventory model of deteriorating items with some constraints in an intuitionistic fuzzy environment. Nagoorgani, A., and Maheswari S., [13] discussed about Vendor-buyer fuzzy inventory system with transportation cost. Abd EI-Wahed, W.F,, Lee, S.M [1] derived some ideas about Interactive fuzzy goal programming for multi-objective transportation problem. Maheswari,S.,and Theresaselas,S.,(2017) [14] discussed about the Safe Transportation of Inventories in Fuzzy Environment. Aggarwal, K.K., Aggarwal, S.P. and Jaggi,C.K.,[2] derived the Impact of Inflation and Credit Policies on Economic Ordering.

An inventory is the stock or the store of an item or resource used by an organization. Inventory means all the materials, parts, supplies, equipments, tools and in process or finished products recorded and kept in organization for some period of time. Inventory is an essential part of every organization/business /manufacturing unit. The organization's inventory management system must carry out objectives set by upper management and must perform in such a way that the organization's profit or performance is enhanced. The objectives set by management will frequently fall in to either of the two categories:

i. Customer service objectives and
ii. Inventory investment objectives

Maintaining inventories is necessary because of the following reasons:

i. Inventory provides service to the customers immediate or at a short notice

ii. Due to absence of stock, the company may have to pay high prices because of piece-wise purchasing. Maintaining of inventory may earn price discount because of bulk purchasing.
iii. Inventory also acts as a buffer stock when raw-materials are received late and so many orders are likely to be rejected.

To earn the goodwill of customers, the manufacturers are expected to supply all items of good quality. However imperfect items are inherently present. This may be due to imperfect production process, natural disasters, damage or breakage in transit and chemical reaction and a host of others. To rectify this, a screening process is employed to pickup imperfect items.

Salameh and Jaber assumed the defective rate to be a random variable and all the defective items can be shipped in a single lot at a discounted price. H.C. Chang developed EOQ model having imperfect quality without shortages in which demand and defective rate are taken as triangular fuzzy numbers. H.M. Wee et al developed an inventory model with imperfect quality allowing shortages, taking defective rate as a random variable. In this paper, an inventory model having imperfect items with shortages is considered where demand, defective rate, holding cost, ordering cost and shortage cost are taken as triangular fuzzy numbers. The supply chain coordination becomes highly effective when the flow of material, information and capital between suppliers are properly taken care of. Many researchers in supply chain management only consider the inventory cost as a criterion but the transportation cost is a major factor which affects the shipment size.

In this paper transportation cost is included in the total cost.Lucio Zavenella, Simone Zanoni investigated single vendor-multiuser environment where the CS policy may be implemented in supply chains. The ultimate end users will be highly satisfied if they receive all good quality items. Valentini et al conducted industrial case study on Consignment Stock of inventories. This paper discusses inspection cost so that the customer's satisfaction is guaranteed with an additional cost which maybe very small.

DEFINITIONS AND PRELIMINARIES ON FUZZY AND INTUITIONISTIC FUZZY SETS

Fuzzy Numbers

A fuzzy subset $\bar{A}$ of the real line R with membership function

$\mu_{\bar{A}}(x)$: $R \rightarrow [0, 1]$ is called a fuzzy number if

i. $\bar{A}$ is normal, (i.e.) there exist an element x_0

Such that $\mu_A(x_0) = 1$

ii) $\bar{A}$ is fuzzy convex,

$$\mu_{\bar{A}}[\lambda x_1 + (1-\lambda) x_2] \geq \mu_{\bar{A}}(x_1) \wedge \mu_{\bar{A}}(x_2),\ x_1, x_2 \in R$$

iii) $\mu_{\bar{A}}(x)$ is upper continuous

iv) Supp $\bar{A}$ is bounded, where supp $\bar{A} = \{x \in R : \mu_{\bar{A}}(x) > 0\}$

Triangular Fuzzy Number:

A fuzzy number $\bar{A}$ of the universe of discourse U may be characterized by a triangular distribution function parameterized by a triplet (a_1,a_2,a_3)

The membership function of the fuzzy number A is defined as,

$$\mu_{\bar{A}}(x) = \begin{cases} 0, & x < a_1 \\ \frac{(x-a_1)}{(a_2-a_1)}, & a_1 < x \leq a_2 \\ \frac{(a_3-x)}{(x-a_2)}, & a_2 < x \leq a_3 \\ 0, & x < a_3 \end{cases}$$

Arithmetic operations of fuzzy numbers

Suppose $\bar{A} = (a_1,a_2,a_3)$and $\bar{B} = (b_1,b_2,b_3)$ are two triangular fuzzy numbers

(i) $\bar{A} + \bar{B} = (a_1,a_2,a_3) + (b_1,b_2,b_3) = (a_1+ b_1,\ a_2+ b_2,\ a_3+ b_3)$

(ii) $\bar{A} \times \bar{B} = (a_1,a_2,a_3) \times (b_1,b_2,b_3) = (a_1b_1,\ a_2b_2,\ a_3b_3)$

(iii) $-\bar{B} = (-b_3,-b_2,-b_1)$ then the subtraction of $\bar{B}$ from $\bar{A}$ is

$\bar{A} - \bar{B} = (a_1,a_2,a_3) - (b_1,b_2,b_3) = (a_1 - b_3,\ a_2 - b_2,\ a_3 - b_1)$

iv) $\frac{1}{\bar{B}} = \bar{B}^{-1} = (\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1})$

$\bar{A} \div \bar{B} = (a_1,a_2,a_3) \div (b_1,b_2,b_3) = (a_1/b_3, a_2/b_2, a_3/b_1)$

v) If a€ R *then* $\alpha\bar{A} = (\alpha a_1, \alpha a_2, \alpha a_3)$ if $\alpha \geq 0$

$$\alpha\bar{A} = (\alpha a_3, \alpha a_2, \alpha a_1) \text{ if } \alpha \leq 0$$

Intuitionistic Fuzzy Set: Let a set X be fixed. An IFS $\tilde{A}$ in X is an object having the form $\tilde{A}$= {<x, $\mu_{\tilde{A}}(x)$, $\upsilon_{\tilde{A}}(x)$>: x∈X}, where the $\mu_{\tilde{A}}(x)$: X): $X \rightarrow [0, 1]$ [0, 1] and $\upsilon_{\tilde{A}}(x)$:): $X \rightarrow [0, 1]$ define the degree of membership and degree of non-membership respectively of the element x∈X to the set $\tilde{A}$,which is a subset of the set X, for every element of x∈X,$0 \leq \mu_{\tilde{A}}(x) + \upsilon_{\tilde{A}}(x) \leq 1$

Intuitionistic Fuzzy Number

Intuitionistic Fuzzy Number: An IFN is defined as $\tilde{A}$ is defined as follows:

$\upsilon_{\tilde{A}}(\lambda x_1 + (1-\lambda) x_2) \leq \text{Max}(\upsilon_{\tilde{A}}(x_1), \upsilon_{\tilde{A}}(x_2))$ ¥ $x_1, x_2 \in R$, $\lambda \in [0,1]$

A triangular intuitionistic fuzzy number:

$\tilde{A} = (a_1,a_2,a_3; w_{\tilde{a}}, u_{\tilde{a}})$ is a subset of intuitionistic fuzzy set on the set of real number R whose membership and non-membership are defined as follows:

$$\begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 < x \leq a_2 \\ \frac{(a_3-x)}{(x-a_2)}, & a_2 < x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \qquad \frac{(a_2-x)}{(a_1'-a_2)}, \quad a_1' < x \leq a_2; \qquad \frac{(x-a_2)}{(a_3'-a_2)}, \quad a_3' < x \leq a_2; \qquad 1, \quad \text{otherwise}$$

Arithmetic Operations of Triangular Intuitionistic Fuzzy Number

If $\tilde{A}$= (a_1, a_2, a_3) (a_1',a_2, a_3') and $\tilde{B}$ = (b_1,b_2,b_3) (b_1',b_2, b_3') are two TIFNs, then

i) Addition of two TIFN is

$\tilde{A} + \tilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3)(a_1'+b_1', a_2+b_2, a_3'+b_3')$ is also TIFN.

ii) Subtraction of two TIFN

$\tilde{A} - \tilde{B} = (a_1-b_3, a_2-b_2, a_3-b_1)(a_1'-b_3', a_2-b_2, a_3'-b_1')$ is also TIFN.

iii) Multiplication of two TIFN

$\tilde{A} \times \tilde{B} = (a_1b_1, a_2b_2, a_3b_3)(a_1'b_1', a_2b_2, a_3'b_3')$ is also TIFN.

iv) If TIFN $\tilde{A}$ = (a_1,a_2,a_3) (a_1', a_2, a_3') and y= ka (with k>0) then $\tilde{y} = k\tilde{A}$

v) Division of two TIFN is

$\frac{\tilde{A}}{\tilde{B}} = (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1})(\frac{a_1'}{b_3'}, \frac{a_2}{b_2}, \frac{a_3'}{b_1'})$ is also TIFN.

Defuzzification for Triangular Fuzzy Number

The defuzzification value for a triangular fuzzy number (a_1, a_2, a_3) is given by

$$\frac{a_1+2a_1+a_3}{4}$$

Defuzzification for Triangular Intuitionistic Fuzzy Number

Let $\breve{A}$ = (a_1, a_2, a_3) (a_1', a_2, a_3') be a triangular intuitionistic fuzzy number. Then the signed distance of can be calculated as follows

$=\frac{1}{4}[\int_1^0 L_\mu(\alpha) + \int_1^0 L_\mu(\alpha) + \int_1^0 L_\mu(\alpha) + \int_1^0 L_\mu(\alpha)]$

$= \frac{1}{4}\int_0^1 [\{a_1 - \alpha(a_2 - a_1)\}\delta\alpha + \int_0^1 \{a_3 - \alpha(a_3 - a_2)\}\delta\alpha + \int_0^1 \{a_2 - (1-\alpha)(a_2 - a_1')\}\delta\alpha + \int_0^1 \{a_2 + (1-\alpha)(a_3' - a_2)\}\delta\alpha]$

$$= \frac{a_1 + 2a_2 + a_3 + a_1' + 2a_2 + a_3'}{8}$$

Model Formulation in Crisp Environment
Assumptions

i) Only single order is produced at the beginning of each cycle

ii) S is the shortage cost per unit quantity and c_0 is the ordering cost per order known as constant

iii) t is the transportation cost and i is the inspection cost are taken in account

iv) T is the cycle length, where as t_1 is the period with no shortage.

v) Where holding cost, inspection cost and transportation cost are taken as an intuitionistic triangular fuzzy numbers.

Notations

The following notations are used for developing the model

h: holding cost per unit quantity per unit time

s: shortage cost per unit quantity per unit time

i: inspection cost per unit quantity per unit time

t: transportation cost per unit quantity per unit time

$c_{0:}$ setup cost per order

T: cycle length

D: total demand

TC: total cost

$S_{a:}$ initial inventory level after fulfilling the backlogged quantity of previous cycle

$S_{b:}$ initial inventory level after fulfilling the backlogged quantity of previous cycle using triangular intuitionistic fuzzy numbers

Q : lot size per cycle

$\bar{h}$: Triangular fuzzy holding cost per unit quantity per unit time

$\bar{s}$: Triangular fuzzy shortage cost per unit quantity per unit time

$\bar{\imath}$: Triangular fuzzy inspection cost per unit quantity per unit time

$\bar{t}$: Triangular fuzzy transportation cost per unit quantity per unit time

$\tilde{h}$: Triangular intuitionstic fuzzy holding cost per unit quantity per unit time

$\tilde{s}$: Triangular intuitionistic fuzzy shortage cost per unit quantity per unit time

$\tilde{\imath}$: Triangular intuitionistic fuzzy inspection cost per unit quantity per unit time

$\tilde{t}$: Triangular intuitionstic fuzzy transportation cost per unit quantity per unit time

$\overline{TC}$: Fuzzy total cost

F(q) : defuzzified total cost

Q: order quantity

q' : fuzzy optimal order quantity

q'' : intuitionistic fuzzy optimal order quantity

F(q') : Minimum defuzzified total cost

F(q'') : minimum intuitionistic defuzzified total cost

T is constant, that is inventory is to be replenished after every time period T. As t_1 is the no shortage period.

$$S_a = D\,t_1$$

$$t_1 = \frac{S_a}{D}$$

Inventory carrying cost during the period [0, t_1]

$$h = \frac{h\,S_a D S_a}{2D} = \frac{h\,s_a^2}{2}$$

Shortage cost during the interval [t_1,T] is

$$S = \frac{1}{2}\ S\,(Q\text{-}s_a)\,(T - t_1)$$

$$= \frac{1}{2D}\,S\,(Q\text{-}s_a)^2 \quad [T - t_1 = \frac{Q - s_a}{D}]$$

Total average of the system is,

$$TC = [\frac{1}{2D}\,S\,(Q\text{-}s_a)^2 + \frac{h\,s_a^2}{2} + \frac{D}{Q}\ i + \frac{D}{Q}\ t]/T$$

The setup cost C_0 and time period T is constant. The average setup cost $\frac{C_0}{T}$ also constant.

The total fuzzy total cost is given by,

Q= DT is constant. We can minimize the above expression with respect to Sa.

Therefore differentiating the total cost w.r.to Sa and equating to zero, We get,

$$S_a = \frac{SQ}{h+s} = \frac{SDT}{h+s}$$

$$C_{min} = \frac{ShQ}{hS} + \frac{D}{Q}\ i + \frac{D}{Q}\ t$$

Fuzzy model with shortages

Let $\bar{h}$ = (h_1,h_2,h_3) , $\bar{s}$ =(s_1,s_2,s_3) , $\bar{\imath}$= (i_1,i_2,i_3) , $\bar{t}$= (t_1,t_2,t_3)

$$\overline{TC} = \frac{1}{T}\ [\frac{\bar{h}s_a^2}{2D} + \frac{1}{2D}\bar{s}\,(Q - s_a^2) + \frac{D}{Q}\bar{\imath} + \frac{D}{Q}\ \bar{t}\ ]$$

$$\overline{TC} = [\frac{(h_1,h_2,h_3)s_a^2}{2D} + \frac{1}{2D}(s_1,s_2,s_3)(Q - s_a^2) + \frac{D}{Q}(i_1,i_2,i_3) + \frac{D}{Q}\ t_1,t_2,t_3)$$

$$\overline{TC} = [\frac{h_1 s_a^2}{2D} + \frac{1}{2D}s_1(Q - s_a^2) + \frac{D}{Q}i_1 + \frac{D}{Q}\ t_1, \frac{h_2 s_a^2}{2D} + \frac{1}{2D}s_2(Q - s_a^2) + \frac{D}{Q}i_2 + \frac{D}{Q}\ t_2, \frac{h_3 s_a^2}{2D} + \frac{1}{2D}s_3(Q - s_a^2) + \frac{D}{Q}i_3 + \frac{D}{Q}\ t_3\ ]$$

Using the distance method for defuzzification of triangular fuzzy number,

$$F(q^*) = \frac{1}{4}\ [(\frac{h_1 s_a^2}{2D} + \frac{1}{2D} s_1(Q - s_a^2) + \frac{D}{Q} i_1 + \frac{D}{Q}\ t_1, \frac{h_2 s_a^2}{2D} + \frac{1}{2D} s_2(Q - s_a^2) + \frac{D}{Q} i_2 + \frac{D}{Q}\ t_2,$$

$$\frac{h_3 s_a^2}{2D} + \frac{1}{2D} s_3(Q - s_a^2) + \frac{D}{Q} i_3 + \frac{D}{Q}\ t_3)\ ]$$

Differentiating F(q) with respect to S_a and equating to zero, we get

$$S_b = \frac{(S_1 + 2S_2 + S_3)Q}{h_1 + S_1 + 2(h_2 + S_2) + h_3 + s_3} = \frac{(S_1 + 2S_2 + S_3)DT}{h_1 + S_1 + 2(h_2 + S_2) + h_3 + s_3}$$

$$\frac{\delta^2 F(q)}{\delta S_1} > 0$$

The minimum cost is given by, where $S_a = S_b$,

$$F(q^{"}) = \frac{1}{4}\ [(\frac{h_1 s_b^2}{2D} + \frac{1}{2D} s_1\left(Q - s_b^2\right) + \frac{D}{Q} i_1 + \frac{D}{Q}\ t_1, \frac{h_2 s_b^2}{2D} + \frac{1}{2D} s_2\left(Q - s_b^2\right) + \frac{D}{Q} i_2 + \frac{D}{Q}\ t_2$$

$$\frac{h_3 s_b^2}{2D} + \frac{1}{2D} s_3\left(Q - s_b^2\right) + \frac{D}{Q} i_3 + \frac{D}{Q}\ t_3)\ ]$$

Intuitionistic fuzzy model with shortages

$\tilde{h}$= (h_1, h_2, h_3) (h_1', h_2, h_3'), $\tilde{S}$ = (s_1, s_2, s_3) (s_1', s_2, s_3'), $\tilde{\imath}$ = (i_1, i_2, i_3) (i_1', i_2, i_3'), $\tilde{t}$= (t_1, t_2, t_3) (t_1', t_2, t_3')

The intuitionstic fuzzy total cost is given by,

$$\widetilde{TC} = \frac{1}{T}\ [\frac{\tilde{h} s_a^2}{2D} + \frac{1}{2D} \tilde{s}\ \ (Q - s_a^2) + \frac{D}{Q} \tilde{\imath}\ + \frac{D}{Q}\ \tilde{t}]$$

$$\widetilde{TC} = \ [\frac{(h_1,h_2,h_3)(h_1',h_2,h_3')s_a^2}{2D} + \frac{1}{2D}(s_1,s_2,s_3)\ \left(s_1',s_2,s_3'\right)(Q - s_a^2) + \frac{D}{Q}(i_1,i_2,i_3)\ \left(i_1',i_2,i_3'\right) + \frac{D}{Q}\ (t_1,t_2,t_3)\left(t_1' t_2,t_3'\right)\ ]$$

$$\widetilde{TC} = \ [\frac{h_1 s_a^2}{2D} + \frac{1}{2D} s_1(Q - s_a^2) + \frac{D}{Q} i_1 + \frac{D}{Q}\ t_1, \frac{h_2 s_a^2}{2D} + \frac{1}{2D} s_2(Q - s_a^2) + \frac{D}{Q} i_2 + \frac{D}{Q}\ t_2, \quad \frac{h_3 s_a^2}{2D} + \frac{1}{2D} s_3(Q - s_a^2) + \frac{D}{Q} i_3 + \frac{D}{Q}\ t_3, \frac{h_1' s_a^2}{2D} + \frac{1}{2D} s_1'(Q - s_a^2) + \frac{D}{Q} i_1' + \frac{D}{Q} t_1', \frac{h_2 s_a^2}{2D} + \frac{1}{2D} s_2(Q - s_a^2) + \frac{D}{Q} i_2 + \frac{D}{Q}\ t_2, \frac{h_3' s_a^2}{2D} + \frac{1}{2D} s_3'(Q - s_a^2) + \frac{D}{Q} i_3' + \frac{D}{Q}\ t_3'$$

Using the signed distance for defuzzification of triangular intuitionstic fuzzy number,

$$F(q') = \frac{1}{8}\ \ [\frac{h_1 s_a^2}{2D} + \frac{1}{2D} s_1(Q - s_a^2) + \frac{D}{Q} i_1 + \frac{D}{Q}\ t_1, \frac{h_2 s_a^2}{2D} + \frac{1}{2D} s_2(Q - s_a^2) + \frac{D}{Q} i_2 + \frac{D}{Q}\ t_2, \quad \frac{h_3 s_a^2}{2D} + \frac{1}{2D} s_3(Q - s_a^2) + \frac{D}{Q} i_3 + \frac{D}{Q}\ t_3, \frac{h_1' s_a^2}{2D} + \frac{1}{2D} s_1'(Q - s_a^2) + \frac{D}{Q} i_1' + \frac{D}{Q} t_1', \frac{h_2 s_a^2}{2D} + \frac{1}{2D} s_2(Q - s_a^2) + \frac{D}{Q} i_2 + \frac{D}{Q}\ t_2, \frac{h_3' s_a^2}{2D} + \frac{1}{2D} s_3'(Q - s_a^2) + \frac{D}{Q} i_3' + \frac{D}{Q}\ t_3'$$

Therefore the optimal shortage quantity and optimal cost is,

$$S_b = \frac{(S_1 + 2S_2 + S_3 + S_1' + 2S_2 + S_1' + S_3')Q}{h_1 + S_1 + 2(h_2 + S_2) + h_3 + s_3 + h_1' + S_1' + 2(h_2 + S_2) + h_3' + S_3'}$$

$$S_b = \frac{(S_1 + 2S_2 + S_3 + S_1' + 2S_2 + S_1' + S_3')DT}{h_1 + S_1 + 2(h_2 + S_2) + h_3 + s_3 + h_1' + S_1' + 2(h_2 + S_2) + h_3' + S_3'}$$

$$F(q'') = \frac{1}{8}\ [\frac{h_1 s_b^2}{2D} + \frac{1}{2D} s_1\left(Q - s_b^2\right) + \frac{D}{Q} i_1 + \frac{D}{Q}\ t_1, \frac{h_2 s_b^2}{2D} + \frac{1}{2D} s_2\left(Q - s_b^2\right) + \frac{D}{Q} i_2 + \frac{D}{Q}\ t_2,\ \ \frac{h_3 s_b^2}{2D} + \frac{1}{2D} s_3\left(Q - s_b^2\right) + \frac{D}{Q} i_3 + \frac{D}{Q}\ t_3, \frac{h_1' s_b^2}{2D} + \frac{1}{2D} s_1'\left(Q - s_b^2\right) + \frac{D}{Q} i_1' + \frac{D}{Q} t_1', \frac{h_2 s_b^2}{2D} + \frac{1}{2D} s_2\left(Q - s_b^2\right) + \frac{D}{Q} i_2 + \frac{D}{Q}\ t_2, \frac{h_3' s_b^2}{2D} + \frac{1}{2D} s_3'\left(Q - s_b^2\right) + \frac{D}{Q} i_3' + \frac{D}{Q}\ t_3'$$

NUMERICAL EXAMPLE

The demand for an item in a company is 10 units per day. The cost of set up is Rs.50 and the holding cost of per day is Rs.10, if shortage cost of 1 unit is Rs.2 if it is late for missing the schedule delivery date. The inspection cost and transportation cost are Rs.5 and Rs.30 respectively. Find the optimal level of inventory per annum.

Solution: Demand D = 10 Units per day

Holding Cost h = Rs. 10/day

Shortage Cost S= Rs. 20/day

Inspection Cost i = Rs. 5/ day

Transportation Cost t = Rs. 30 / day

The optimal inventory level is,

S_a = 2.49629

The minimum total cost is C_{min} = 98.14625

Q^0 = 3.7444, S_a = 21.34. Here $Q^0 > Sa$

Fuzzy case (TFN)

D = 10 Units / day

$\bar{h}$ = (8,10,13)

$\bar{s}$ = (19,20,22)

$\bar{\iota}$ = (3,5,6.8)

$\bar{t}$ = (27.5,30,33)

S_b = 2.47082

F(q') = 99.00170

Intuitionistic fuzzy case (TIFN)

D=10 Units/day

$\tilde{h}$ = (8,10,13)(7,10,14)

$\tilde{s}$ = (19,20,22) (17.5,20,23)

$\tilde{\iota}$ = (3,5,6.8) (2,5,7.5)

$\tilde{t}$ = (27.5,30,33) (25.5,30,34)

S_b = 2.4652

F(q'') = 98.62153

Sensitivity Analysis when associated costs are TFN and TIFN

FUZZY ENVIRONMENT							
D	(h_1, h_2, h_3)	(s_1, s_2, s_3)	(i_1, i_2, i_3)	$t_1, t_2, t_3)$	q'	S_b	F(q')
20	**(8,10,13)**	(19,20,22)	(3,5,6.8)	(27.5,30,33)	4.68878	3.11304	**152.97088**
5	**(8,10,13)**	(19,20,22)	(3,5,6.8)	(27.5,30,33)	2.95374	1.69109	**65.31139**
10	**(10.5,12,15.2)**	(19,20,22)	(3,5,6.8)	(27.5,30,33)	3.57134	2.2130	**103.12296**
10	**(6.5,8,11.2)**	(19,20,22)	(3,5,6.8)	(27.5,30,33)	3.89196	2.74846	**94.62777**

10	**(8,10,13)**	(19,20,22)	**(6.5,8,11.2)**	(27.5,30,33)	3.84053	2.54986	**105.39567**
10	**(8,10,13)**	(19,20,22)	**(3.5,4,6)**	(27.5,30,33)	3.70104	2.45725	**98.18548**
10	**(8,10,13)**	(19,20,22)	(3,5,6.8)	**(31.5,32,34)**	**3.79942**	**2.52257**	**103.15068**
10	**(8,10,13)**	(19,20,22)	(3,5,6.8)	**(27.5,28,30)**	**3.65853**	**2.42902**	**92.64290**
INTUTIONISTIC FUZZY ENVIRONMENT							
D	(h_1, h_2, h_3) (h'_1, h_2, h'_3)	(s_1, s_2, s_3) (s'_1, s_2, s'_3)	(i_1, i_2, i_3) (i'_1, i_2, i'_3)	$t_1, t_2, t_3)$ (t'_1, t_2, t'_3)	q"	S_b	$F(q)''$
20	**(8,10,13)** (7,10,14)	(19,20,22) (17.5,20,23)	(3,5,6.8) (2,5,7.5)	(27.5,30,33) (25.5,30,34)	4.68315	3.10607	**152.82583**
5	**(8,10,13)** (7,10,14)	(19,20,22) (17.5,20,23)	(3,5,6.8) (2,5,7.5)	(27.5,30,33) (25.5,30,34)	2.95020	1.95670	**65.08663**
10	**(10.5,12,15.2)** **(8.5,12,17.1)**	(19,20,22) (17.5,20,23)	(3,5,6.8) (2,5,7.5)	(27.5,30,33) (25.5,30,34)	3.56795	2.20945	**101.27838**
10	**(6.5,8,11.2)** **(4.5,8,13.1)**	(19,20,22) (17.5,20,23)	(3,5,6.8) (2,5,7.5)	(27.5,30,33) (25.5,30,34)	3.88848	2.74471	**94.27368**
10	**(8,10,13)** (7,10,14)	(19,20,22) (17.5,20,23)	**(6.5,8,11.2)** **(4.5,8,13.1)**	(27.5,30,33) (25.5,30,34)	3.83729	2.54506	**105.10848**
10	**(8,10,13)** (7,10,14)	(19,20,22) (17.5,20,23)	**(3.5,4,6)** **(1,4,6.5)**	(27.5,30,33) (25.5,30,34)	3.68886	2.44662	**97.13359**
10	**(8,10,13)** (7,10,14)	(19,20,22) (17.5,20,23)	(3,5,6.8) (2,5,7.5)	**(31.5,32,34)** **(29,32,34.5)**	3.78967	2.51348	**101.51059**
10	**(8,10,13)** (7,10,14)	(19,20,22) (17.5,20,23)	(3,5,6.8) (2,5,7.5)	**(27.5,28,30)** **(25,28,30.5)**	3.64925	2.42034	**95.05774**

From the sensitivity analysis,

- If the **holding cost** increases then the optimal fuzzy and intuitionstic fuzzy optimal order quantity increase, if it decreases then the optimal fuzzy and intuitionstic fuzzy optimal order quantity decrease.

- If the **inspection cost** increases then the optimal fuzzy and intuitionstic fuzzy optimal order quantity increase, if it decreases then the optimal fuzzy and intuitionstic fuzzy optimal order quantity decrease.

- If the **transportation cost** increases then the optimal fuzzy and intuitionstic fuzzy optimal order quantity increase, if it decreases then the optimal fuzzy and intuitionstic fuzzy optimal order quantity decrease.

Conclusion

From the analysis of the problem under fuzzy and intuitionistic environment has shown that the shortage quantity obtained under the intuitionistic environment is closer to the fuzzy quantity. Therefore, when membership function is not always accurately defined to the lack of personal error, an intuitionistic fuzzy set may help in solving the problem. The relative size of h and s has an influence of shortages. For large h relative to s the effect on Q is considerable. Holding cost is small relative to s minor changes in quantity and cost can be expected. For very large shortage cost the backorder model is the basic EOQ model without shortages. From the sensitivity analysis, if the **holding cost** increases then the optimal fuzzy and intuitionstic fuzzy optimal order quantity increase, if it decreases then the optimal fuzzy and intuitionstic fuzzy optimal order quantity decrease. If the **inspection cost** increases then the optimal fuzzy and intuitionstic fuzzy optimal order quantity increase, if it decreases then the optimal fuzzy and intuitionstic fuzzy optimal order quantity decrease .If the **transportation cost** increases then the optimal fuzzy and intuitionstic fuzzy optimal order quantity increase, if it decreases then the optimal fuzzy and intuitionstic fuzzy optimal order quantity decrease.

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