Upper and Lower Bound Integer Transportation Problem of Fuzzy Interval Integer Transportation Problem with Mixed Constraints

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The upper bound integer transportation problem of the fuzzy interval integer transportation problem is

$$\text{min. } z_2 = \sum_{i=1}^{m} \square \sum_{j=1}^{n} c_{ij}^{2} x_{ij}^{2}$$

s.t.

$$\sum_{j=1}^{n} x_{ij}^{2} \approx a_i^{2},\ i = 1, 2, 3, \ldots, m\ ;$$

$$\sum_{j=1}^{n} x_{ij}^{2} \geq a_i^{2},\ i = 1, 2, 3, \ldots, m\ ;$$

$$\sum_{j=1}^{n} x_{ij}^{2} \leq a_i^{2},\ i = 1, 2, 3, \ldots, m\ ;$$

$$\sum_{i=1}^{m} x_{ij}^{2} \approx b_j^{2},\ j = 1, 2, 3, \ldots, n\ ;$$

$$\sum_{i=1}^{m} x_{ij}^{2} \geq b_j^{2},\ j = 1, 2, 3, \ldots, n\ ;$$

$$\sum_{i=1}^{m} x_{ij}^{2} \leq b_j^{2},\ j = 1, 2, 3, \ldots, n\ ;$$

Then the set $\{\bar{x}_{ij}^{2}$for all i and j$\}$ is an optimal solution of the upper bound integer transportation problem. The lower bound integer transportation problem of the fuzzy interval integer transportation problem is

$$\text{min. } z_1 = \sum_{i=1}^{m} \square \sum_{j=1}^{n} c_{ij}^{1} x_{ij}^{1}$$

s.t.

$$\sum_{j=1}^{n} x_{ij}^{1} \approx a_i^{1},\ i = 1, 2, 3, \ldots, m\ ;$$

$$\sum_{j=1}^{n} x_{ij}^{1} \geq a_i^{1},\ i = 1, 2, 3, \ldots, m\ ;$$

$$\sum_{j=1}^{n} x_{ij}^{1} \leq a_i^{1},\ i = 1, 2, 3, \ldots, m\ ;$$

$$\sum_{i=1}^{m} x_{ij}^{1} \approx b_{j}^{1}, j = 1, 2, 3, \ldots, n \,;$$
$$\sum_{i=1}^{m} x_{ij}^{1} \geq b_{j}^{1}, j = 1, 2, 3, \ldots, n \,;$$
$$\sum_{i=1}^{m} x_{ij}^{1} \leq b_{j}^{1}, j = 1, 2, 3, \ldots, n \,;$$

Then the set $\{\bar{x}_{ij}^{1}$ for all i and j$\}$ is an optimal solution of the lower bound integer transportation problem.

Separation Method

Separation method can be understood with the help of algorithm for solving fuzzy interval integer transportation problem. Algorithm of the separation method is as follows.

Step 1: Write the upper bound integer transportation problem of the given fuzzy interval integer transportation problem.

Step 2: Solve the upper bound integer transportation problem using zero method.

Step 3: Construct the lower bound integer transportation problem of the given fuzzy interval integer transportation problem.

Step 4: Solve the lower bound integer transportation problem using zero method.

Step 5: The solution of the given fuzzy interval integer transportation problem is $\{[\bar{x}_{ij}^{1}, \bar{x}_{ij}^{2}]$, for all i and j$\}$.

Zero Method

We, now proposed a new method called zero method for finding the optimal solution for the transportation problem. The method is proceeding as follows.

Step 1: Convert all inequalities into equalities.

Step 2: If the given transportation is in unbalanced transportation problem, then make it balance transportation problem by introducing dummy rows or columns.

Step 3: Subtract smallest row element of the transportation table from the corresponding row entries.

Step 4: Subtract smallest column element of the transportation table from the corresponding column entries.

Step 5: Remember that each row and each column has at least one zero.

Step 6: Allocate the minimum cost from demand or supply in the corresponding zero.

Step 7: Repeat the procedure form the Step 3 to Step 6, until we get the optimal solution.

Step 8: Place the loads of the dummy rows or columns of the balanced at the lowest cost feasible cells of the given transportation problem to obtain the optimal solution for the transportation problem with mixed constraints.

Step 9: Thus we get the optimal solution for the new transportation problem with mixed constraints.

Numerical Example

Consider the following fuzzy integer transportation problem with mixed constraints.

Table 1

	1	**2**	**3**	**Supply**
1	(1, 2, 3, 4)	(2, 5, 8, 11)	(2, 4, 6, 8)	≈ (2, 5, 8, 11)
2	(2, 6, 10, 14)	(1, 3, 5, 7)	(0, 1, 2, 3)	≥ (3, 6, 9, 12)
3	(4, 8, 12, 16)	(3, 9, 15, 21)	(1, 2, 3, 4)	≤ (3, 9, 15, 21)
Demand	≈ (4, 8, 12, 16)	≥ (8, 10, 12, 14)	≤ (3, 5, 7, 9)	

Solution: Now, the fuzzy interval integer transportation problem of the above problem is given below.

Table 2

	1	**2**	**3**	**Supply**
1	$(1+\alpha, 4-\alpha)$	$(2+3\alpha, 11-3\alpha)$	$(2+2\alpha, 8-2\alpha)$	$\approx (2+3\alpha, 11-3\alpha)$
2	$(2+4\alpha, 14-4\alpha)$	$(1+2\alpha, 7-2\alpha)$	$(0+\alpha, 3-\alpha)$	$\geq (3+3\alpha, 12-3\alpha)$
3	$(4+4\alpha, 16-4\alpha)$	$(3+6\alpha, 21-6\alpha)$	$(1+\alpha, 4-\alpha)$	$\leq (3+6\alpha, 21-6\alpha)$
Demand	$\approx (4+4\alpha, 16-4\alpha)$	$\geq (8+2\alpha, 14-2\alpha)$	$\leq (3+2\alpha, 9-2\alpha)$	

Put $\alpha = 0$ in the above fuzzy interval integer transportation problem. We get the following fuzzy interval integer transportation problem with variables $\left[x_{ij}^{1}, x_{ij}^{4}\right]$ for all i and j corresponding to the above interval integer transportation problem.

Table 3

	1	**2**	**3**	**Supply**
1	(1, 4)	(2, 11)	(2, 8)	≈ (2, 11)
2	(2, 14)	(1, 7)	(0, 3)	≥ (3, 12)
3	(4, 16)	(3, 21)	(1, 4)	≤ (3, 21)
Demand	≈ (4, 16)	≥ (8, 14)	≤ (3, 9)	

Now, the upper bound integer transportation problem of the above fuzzy interval integer transportation problem is as follow.

Table 4

	1	**2**	**3**	**Supply**
1	4	11	8	≈ 11
2	14	7	3	≥ 12
3	16	21	4	≤ 21
Demand	≈ 16	≥ 14	≤ 9	

Convert the all inequalities into equalities; we get the following transportation problem

Table 5

	1	**2**	**3**	**Supply**
1	4	11	8	= 11
2	14	7	3	= 12
3	16	21	4	= 21
Demand	= 16	= 14	= 9	

Since, it is not balanced transportation problem. So for make it balanced, we add a dummy column with zero value.

Table 6

	1	**2**	**3**	**4**	**Supply**
1	4	11	8	0	= 11
2	14	7	3	0	= 12
3	16	21	4	0	= 21
Demand	= 16	= 14	= 9	5	= 44

Using Step 3 and Step 4, we get the following transportation table

Table 7

	1	**2**	**3**	**4**	**Supply**
1	0	4	5	0	= 11
2	10	0	0	0	= 12
3	12	14	1	0	= 21
Demand	= 16	= 14	= 9	5	= 44

Now, using the zero method, the optimal solution to the upper bound integer transportation problem is

Table 8

	1	**2**	**3**	**4**	**Supply**
1	0**[6]**	4	5	0**[5]**	= 11
2	10	0**[3]**	0**[9]**	0	= 12
3	12**[10]**	14**[11]**	1	0	= 21
Demand	= 16	= 14	= 9	5	= 44

Now, using the step 8, we get the following solution for the upper bound integer transportation problem.

Table 9

	1	**2**	**3**	**Supply**
1	4**[11]**	11	8	≈ 11
2	14	7**[3]**	3**[9]**	≥ 12
3	16**[10]**	21**[11]**	4	≤ 21
Demand	≈ 16	≥ 14	≤ 9	

So, the optimal solution of upper bound integer transportation problem is
$\bar{x}_{11}^{4}=11, \bar{x}_{22}^{4}=3, \bar{x}_{23}^{4}=9, \bar{x}_{31}^{4}=10, \bar{x}_{32}^{4}=11$ and the transportation cost is min. $z_4 = 483$.

Now, the lower bound integer transportation problem of the above fuzzy interval integer transportation problem is as follow.

Table 10

	1	**2**	**3**	**Supply**
1	1	2	2	≈ 2
2	2	1	0	≥ 3
3	4	3	1	≤ 3
Demand	≈ 4	≥ 8	≤ 3	

Convert the all inequalities into equalities; we get the following transportation problem:

Table 11

	1	**2**	**3**	**Supply**
1	1	2	2	= 2
2	2	1	0	= 3
3	4	3	1	= 3
Demand	= 4	= 8	= 3	

Since, it is not balanced transportation problem. So for make it balanced, we add a dummy row with zero value.

Table 12

	1	**2**	**3**	**Supply**
1	1	2	2	= 2
2	2	1	0	= 3
3	4	3	1	= 3
4	0	0	0	= 7
Demand	= 4	= 8	= 3	= 15

Using Step 3 and Step 4, we get the following transportation table:

Table 13

	1	**2**	**3**	**Supply**
1	0	1	1	= 2
2	2	1	0	= 3
3	3	2	0	= 3
4	0	0	0	= 7
Demand	= 4	= 8	= 3	= 15

Now, using the zero method, the optimal solution to the lower bound integer transportation problem is

Table 14

	1	**2**	**3**	**Supply**
1	0[2]	1	1	= 2
2	2	1	0[3]	= 3
3	3	2[3]	0	= 3
4	0[2]	0[5]	0	= 7
Demand	= 4	= 8	= 3	= 15

Using Step 8, we get the optimal solution of lower bound integer transportation problem is

Table 15

	1	**2**	**3**	**Supply**
1	1**[4]**	2	2	≈ 2
2	2	1**[5]**	0**[3]**	≥ 3
3	4	3**[3]**	1	≤ 3
Demand	≈ 4	≥ 8	≤ 3	

$\bar{x}_{11}^{1}=4, \bar{x}_{22}^{1}=5, \bar{x}_{23}^{1}=3, \bar{x}_{32}^{1}=3$ and the transportation cost is Min. $z_1 = 18$.

Put $\alpha = 1$ in the above fuzzy interval integer transportation problem. We get the following fuzzy interval integer transportation problem with variables $\left[x_{ij}^{2}, x_{ij}^{3}\right]$ for all *i* and *j*.

Table 16

	1	**2**	**3**	**Supply**
1	(2,3)	(5, 8)	(4, 6)	≈ (5, 8)
2	(6,10)	(3, 5)	(1,2)	≥ (6, 9)
3	(8, 12)	(9,15)	(2, 3)	≤ (9,15)
Demand	≈ (8,12)	≥ (10, 12)	≤ (5, 7)	

Now, the upper bound integer transportation problem of the above fuzzy interval integer transportation problem is as follows.

Table 17

	1	**2**	**3**	**Supply**
1	3	8	6	≈ 8
2	10	5	2	≥ 9
3	12	15	3	≤ 15
Demand	≈ 12	≥ 12	≤ 7	

Convert the all inequalities into equalities; we get the following transportation problem

Table 18

	1	**2**	**3**	**Supply**
1	3	8	6	= 8
2	10	5	2	= 9
3	12	15	3	= 15
Demand	= 12	= 12	= 7	

Since, it is not balanced transportation problem. So for make it balanced, we add a dummy column with zero value.

Table 19

	1	**2**	**3**	**4**	**Supply**
1	3	8	6	0	= 8
2	10	5	2	0	= 9
3	12	15	3	0	= 15
Demand	= 12	= 12	= 7	= 1	= 32

Using Step 3 and Step 4, we get the following transportation table:

Table 20

	1	**2**	**3**	**4**	**Supply**
1	0	3	4	0	= 8
2	7	0	0	0	= 9
3	9	10	1	0	= 15
Demand	= 12	= 12	= 7	= 1	= 32

Now, using the zero method, the optimal solution to the lower bound integer transportation problem is

Table 21

	1	**2**	**3**	**4**	**Supply**
1	0[7]	3	4	0[1]	= 8
2	7	0[2]	0[7]	0	= 9
3	9[5]	10[10]	1	0	= 15
Demand	= 12	= 12	= 7	= 1	= 32

Using step 8, we get the following optimal solution for the upper bound integer transportation problem.

Table 22

	1	**2**	**3**	**Supply**
1	3**[8]**	8	6	≈ 8
2	10	5**[2]**	2**[7]**	≥ 9
3	12**[5]**	15**[10]**	3	≤ 15
Demand	≈ 12	≥ 12	≤ 7	

So, the optimal solution of upper bound integer transportation problem is
$\bar{x}_{11}^3=8, \bar{x}_{22}^3=2, \bar{x}_{23}^3=7, \bar{x}_{31}^3=5, \bar{x}_{32}^3=10$ and the transportation cost is min. $z_3 = 258$.

Now, the lower bound integer transportation problem of the above fuzzy interval integer transportation problem is as follows.

Table 23

	1	**2**	**3**	**Supply**
1	2	5	4	≈ 5
2	6	3	1	≥ 6
3	8	9	2	≤ 9
Demand	≈ 8	≥ 10	≤ 5	

Convert the all inequalities into equalities; we get the following transportation problem

Table 24

	1	**2**	**3**	**Supply**
1	2	5	4	= 5
2	6	3	1	= 6
3	8	9	2	= 9
Demand	= 8	= 10	= 5	

Since, it is not balanced transportation problem. So for make it balanced, we add a dummy row with zero value.

Table 25

	1	**2**	**3**	**Supply**
1	2	5	4	= 5
2	6	3	1	= 6
3	8	9	2	= 9
4	0	0	0	= 3
Demand	= 8	= 10	= 5	= 23

Using Step 3 and Step 4, we get the following transportation table:

Table 26

	1	**2**	**3**	**Supply**
1	0	3	2	= 5
2	5	2	0	= 6
3	6	7	0	= 9
4	0	0	0	= 3
Demand	= 8	= 10	= 5	= 23

Now, using the zero method, the optimal solution to the lower bound integer transportation problem is

Table 27

	1	**2**	**3**	**Supply**
1	0[5]	3	2	= 5
2	5	2[4]	0[2]	= 6
3	6[3]	7[6]	0	= 9
4	0	0	0[3]	= 3
Demand	= 8	= 10	= 5	= 23

Using Step 8, we get the optimal solution of lower bound integer transportation problem is

Table 28

	1	**2**	**3**	**Supply**
1	2**[5]**	5	4	≈ 5
2	6	3**[4]**	1**[5]**	≥ 6
3	8**[3]**	9**[6]**	2	≤ 9
Demand	≈ 8	≥ 10	≤ 5	

$\bar{x}_{11}^2 = 5, \bar{x}_{22}^2 = 4, \bar{x}_{23}^2 = 5, \bar{x}_{31}^2 = 3, \bar{x}_{32}^2 = 6$ and the transportation cost is Min. $z_2 = 105$.

Hence, the fuzzy optimal solution for the given fuzzy integer transportation problem is $\tilde{x}_{11} \approx (4, 5, 8, 11)$, $\tilde{x}_{22} \approx (5, 4, 2, 3)$, $\tilde{x}_{23} \approx (3, 5, 7, 9)$, $\tilde{x}_{31} \approx (0, 3, 5, 10)$ and $\tilde{x}_{32} \approx (3, 6, 10, 11)$ with the fuzzy objective value $\tilde{z} = (18, 105, 258, 483)$.

Conclusion

We have attempted to develop the separation method based on zero method provides an optimal solution of the fuzzy interval integer transportation problem with mixed constraints. This method is a systematic procedure, which is very simple, easy to understand and apply. This method provides more options and can be served an important tool for the decision makers when they are handling various types of logistic problems interval parameters.

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