Figuring Volume Using Double Integration to Deal with the Problems of Electronics Engineering

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Abstract:
Despite the great importance of estimating volume bounded by several surfaces in the field of advanced Mathematics and Electronics engineering, it is tough to visualize it three dimensionally and then to figure it out two dimensionally. Thus, the process of achieving the same becomes complicated and tedious. This paper provides a new approach to solve the so called problems of volume by converting them into simple problems of double integration.

Keywords: Volume, Double Integration, Room Impulse Response, Plane.

I. INTRODUCTION

Few of the researchers presented the various approaches for explaining the estimation of volume bounded by a number of surfaces. A Dorko and N M Speer [1] have presented the volume as connection of an array of cubes. S Dinas and J M Banon [2] have shown the bounding volume by using the composition of geometrical shapes.

In acoustic signal processing, many times simulation of room acoustics is required for acoustic eco cancellation, acoustic feedback cancellation and acoustic noise cancellation. Modelling of room impulse response commonly done by three methods, wave based method, ray based method and statistical method. The image source method given by Allen and Barclay in 1976 is most commonly used in category of ray-based method to calculate the room impulse response. In acoustic signal processing, the acoustic characteristics between two points in a room is calculated by just generating an impulse by a sound source and then calculate its amplitude after many reflections from the different walls of the room before it reaches to the microphone. These collections of all amplitude rays is called room impulse response. This room impulse response function behaves as a notch filter and it is capable to reject or pass some frequencies according to the position of notch.

The response of the room impulse response function for basic test signals such as impulse, unit step, exponential and ramp is given by B K Sharma and M Arif [3]. The output characteristics found as low pass filter and minimum phase. Characterization of this room impulse response function for ramp signal is given by B K Sharma and M Arif [4]. The phase characteristics come as minimum phase. Characterization of this function for different width ramp signal is given by B K Sharma and M Arif [5]. The room impulse response depends upon the dimension of the room, reflection coefficients, Order of reflections, temperature and the air density of the room. Basically the room for which room impulse response calculated behave as a cavity and each cavity has its resonance frequency, which is dependent on its shape and volume. If we control the cavity shape and volume, then we can easily find/shift notch of the response to the desired position. So the volume calculation to shift the notch of room impulse response characteristics at the desired position is a helpful parameter.

In this paper, a new approach is presented to solve the above mentioned volume problems. According to this approach, we do not try to create a two dimensional structure in one-one correspondence of the given three dimensional problem. Instead of it, we solve the given problem by converting it directly in a two dimensional problem based over double integration.

II. ALGORITHM

Recently, R. Nelsen [6] evolved an effective technique called "proof without words" based over geometrical analogy of algorithms. The algorithm mentioned here for solving volume problems is somewhere analogues to Nelson's technique.

Here, we use the observational technique to find number of different values for each of the variables x, y and z using all the surfaces given in the problem. If any of these variables has exactly two different values, then we select the appropriate formula for evaluating volume V accordingly from the list of three formulae given below:

$$V = \iint_{A_{xy}} (z_2 - z_1)\, dx\, dy \tag{1}$$

or $$V = \iint_{A_{yz}} (x_2 - x_1)\, dy\, dz \tag{2}$$

or $$V = \iint_{A_{xz}} (y_2 - y_1)\, dx\, dz \tag{3}$$

Above three formulae are expanded forms of the primary formula $V = \iint_D z\ dx\, dy$ mentioned by B V Ramana [7].

After choosing correct formula from the above mentioned list, limits for the respective double integral can be obtained using the relevant plane. For example, if formula (3) is selected for evaluating volume in any problem, then limits for the respective double integral can be obtained in $xz-$plane by substituting $y = 0$ in equations of all the surfaces given.

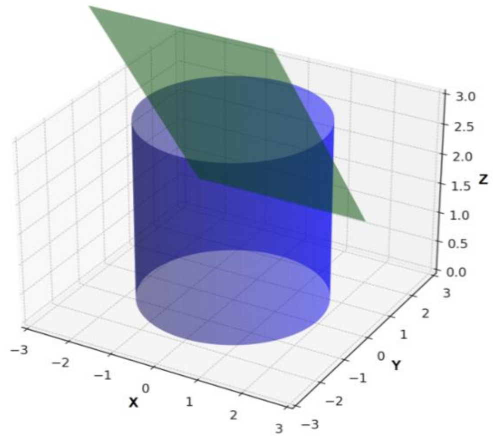
III. EXAMPLES

Example 3.1

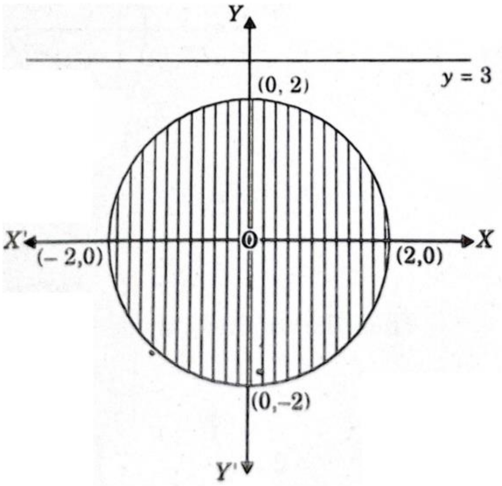
Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 3$ and $z = 0$ using double integration.

Solution.

Initially if we thought about figuring all surfaces given in the problem, then the following three dimensional diagram can be visualized:



Now, it seems complicated to create a mathematical model for volume evaluation by analyzing above three dimensional diagram. So, we use the algorithm mentioned in this paper. Here we observe exactly 2 values for variable x, 3 values for variable y and 2 values for variable z by analyzing all the equations of the surfaces given in the problem. Hence, any of the formula mentioned in equation (1) or equation (3) can be used to find the required volume. If we select equation (1) for the purpose, then two values of variable z are taken as $z_2 = 3 - y$ and $z_1 = 0$. Hence, the required volume is given by $V = \iint_{A_{xy}} (3-y)\,dx\,dy,$ where to find the limits in $xy-$plane, we substitute $z = 0$ in all the equations of given surfaces and obtain $x^2 + y^2 = 4,\ y = 3.$ Common bounded region between these curves (shaded by vertical strips) can be visualized in the following diagram:



Thus, the required volume is finally obtained as:

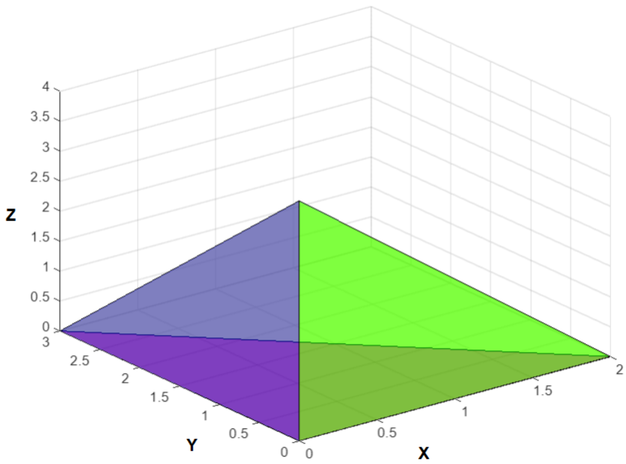
$$V = \int_{x=-2}^{2} \int_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (3-y)\,dx\,dy = \int_{x=-2}^{2} \left(3y - \frac{y^2}{2}\right)_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dx = 12 \int_{x=0}^{2} \sqrt{4-x^2}\,dx = 12\pi$$

Example 3.2

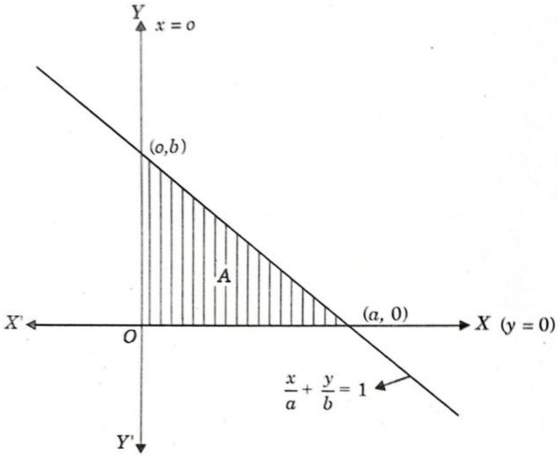
Find the volume bounded by the planes $x = 0, y = 0, z = 0$ *and* $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ using double integration.

Solution.

Initially if we thought about figuring all surfaces given in the problem, then the following three dimensional diagram can be visualized:



Now, it seems complicated to create a mathematical model for volume evaluation by analyzing above three dimensional diagram like previous problem. So, we use the algorithm mentioned in this paper. Here we observe exactly 2 values for variable x, 2 values for variable y and 2 values for variable z by analyzing all the equations of the surfaces given in the problem. Hence, any of the formula mentioned in equation (1) or equation (2) or equation (3) can be used to find the required volume. If we select equation (1) for the purpose, then two values of variable z are taken as $z_2 = 1 - \frac{x}{a} - \frac{y}{b}$ and $z_1 = 0$. Hence, the required volume is given by $V = \iint_{A_{xy}} \left(1 - \frac{x}{a} - \frac{y}{b}\right) dx\,dy,$ where to find the limits in $xy-$plane, we substitute $z = 0$ in all the equations of given surfaces and obtain $x = 0,\ y = 0,\ \frac{x}{a} + \frac{y}{b} = 1.$ Common bounded region between these curves (shaded by vertical strips) can be visualized in the following diagram:



Thus, the required volume is finally obtained as:

$$V = c\int_{x=0}^{a}\int_{y=0}^{b\left(1-\frac{x}{a}\right)} \left(1 - \frac{x}{a} - \frac{y}{b}\right) dx\,dy = c\int_{x=0}^{a} \left(y - \frac{xy}{a} - \frac{y^2}{2b}\right)_{y=0}^{b\left(1-\frac{x}{a}\right)} dx = \frac{bc}{2}\int_{x=0}^{a}\left(1 - \frac{x}{a}\right)^2 dx = \frac{abc}{6}$$

IV. CONCLUSION

After observing above examples, it is obvious to say that instead of solving them three dimensionally, their solutions are obtained in most simple manner by the algorithm presented in this paper.
Thus, it can easily be said that the efficiency and accuracy of the algorithm presented here is very high in comparison of the complicated three dimensional structures and hence above mentioned volume based problem of room impulse response as well as other important volume based problems of Electronics engineering are easily and perfectly solvable by the prescribed algorithm.

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