Graph-Based Routing Algorithms for Enhancing Communication Efficiency

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Abstract

Efficient communication is essential for modern networked systems, impacting applications from data centers and telecommunication networks to sensor and urban traffic networks. Graph-based routing algorithms leverage graph theory to model networks, using nodes as devices or locations and edges as connections to determine optimized routes for data transmission. These algorithms, such as Dijkstra's, Bellman-Ford, and A*, allow for flexible and efficient routing by accounting for factors like network congestion, fault tolerance, shortest path, and load balancing. In varied environmentsfrom static to dynamic networksthese algorithms maximize throughput, reduce latency, and improve resource utilization. Studies have advanced this field by addressing unique challenges, including fault tolerance in wireless sensor networks (WSNs), adaptive routing in cognitive radio networks (CRNs), and routing in delay-sensitive and resource-constrained environments like interplanetary and vehicular ad hoc networks (VANETs). Overall, graph-based routing algorithms provide a robust framework for enhancing communication efficiency, demonstrating their adaptability across diverse network topologies and applications.

Keywords: Network Optimization, Graph Theory, Fault Tolerance, Load Balancing

1. Introduction

Efficient communication in networked systems is a critical requirement across numerous applications, from data centers and telecommunications networks to sensor networks and urban traffic systems. Graph-based routing algorithms have become an essential tool in achieving optimized communication pathways, enabling faster data transfer, improved resource utilization, and reduced latency. With modeling networks as graphswhere nodes represent devices or locations and edges represent connections between themthese algorithms can systematically evaluate multiple routing options, considering factors such as network congestion, shortest path, fault tolerance, and load balancing. The primary objective of graph-based routing algorithms is to enhance communication efficiency, minimizing data travel time and maximizing throughput. Algorithms such as Dijkstra's, Bellman-Ford, and A* have been foundational, each with unique strengths in path optimization depending on network topology and requirements. For example, Dijkstra's algorithm can accommodate negative weights and detect negative cycles. A* combines the strengths of Dijkstra's with heuristic-driven search, often employed in dynamic systems where efficiency is crucial in real-time.

Graph Theory in Network Design

Graph theory provides a mathematical framework for modeling relationships in a network, which makes it

foundational for routing in communication networks. In graph-based routing, nodes represent devices, and edges represent connections or communication paths between them.

Basic Concepts in Graph Theory

- Graph: A graph G = (V, E) consists of a set of vertices (or nodes) V and a set of edges E. Each edge $e \in E$ represents a connection between a pair of nodes.
- **Directed Graph** (**Digraph**): In a directed graph, each edge has a direction, from one node to another. Directed graphs are useful for asymmetric networks where data flow is one-way.
- **Undirected Graph**: In an undirected graph, edges have no direction, meaning data can flow both ways. It's often used to represent symmetric networks.
- Weighted Graph: Each edge in a weighted graph has a weight, representing metrics such as distance, latency, or bandwidth, which is essential for finding the most efficient paths in a network.
- **Path**: A path is a sequence of edges connecting a series of vertices. In network routing, finding the optimal path between nodes is crucial.
- **Shortest Path**: The shortest path is the path between two nodes that minimizes the sum of the edge weights. Algorithms like Dijkstra's or Bellman-Ford are commonly used to find shortest paths for routing purposes.

Applications of Graph Theory in Network Routing

- **Network Modeling**: Graphs are used to model network topologies, representing nodes (routers, switches) and edges (cables, wireless links) for both wired and wireless networks.
- **Routing Optimization**: Using graph algorithms, such as shortest path algorithms, to determine optimal routing paths to minimize latency or maximize bandwidth efficiency.
- **Load Balancing**: With assigning weights based on load, graphs can help in balancing network traffic to prevent congestion and improve overall network performance.
- **Reliability and Fault Tolerance**: Graphs enable analysis of redundancy in paths. In case of a network failure, algorithms can dynamically reroute traffic through alternative paths to maintain connectivity.

2. Related Reviews

Kondareddy and Agrawal (2010) conducted a study focused on improving routing efficiency in cognitive radio networks (CRNs), which enable dynamic spectrum access and feature constantly shifting channel sets at each node. The researchers noted that conventional graph-based routing algorithms, like DSR and AODV, used in fixed-spectrum multi-hop networks, are not suitable for CRNs due to their inability to model the dynamic nature of the channels effectively. The study aimed to introduce a new multi-edge planar graph model that could capture the necessary information for optimal routing in CRNs. The methodology involved developing the model and testing it through simulations to compare its complexity and effectiveness with a previously proposed layered graph model. The findings demonstrated that the multi-edge planar graph model reduced complexity while maintaining performance, making it more suitable for routing in dynamic-spectrum networks. This study is highly relevant to the field of graph-based routing algorithms as it addresses the challenges of dynamic environments, particularly in cognitive radio networks, by proposing a flexible model that can be integrated with existing graph-based routing protocols. Its contribution lies in enhancing communication efficiency in networks with fluctuating channel availability, a common scenario in future wireless and sensor networks.

Mishra, Mathew, et al. (2011) aimed to address the issue of fault susceptibility in dense Wireless Sensor Networks (WSNs), proposing a fault-aware routing solution to maintain network performance despite node failures. Their methodology involved the development of a Multipurpose Architecture and Routing Protocol (MARP) based on a de-Bruijn graph, designed to support multicast routing, broadcasting, fault detection, and fault tolerance. The protocol was structured to be scalable, allowing for straightforward mapping between three-dimensional and two-dimensional spaces. The researchers tested the practicality of the protocol in real-world environments using mica2 motes, demonstrating its effectiveness in managing faults. Their findings indicated that higher redundancy levels were necessary to enhance fault tolerance, ensuring network reliability during node failures. The study is highly relevant to graph-based routing algorithms for enhancing communication efficiency, particularly in fault-tolerant WSNs. By leveraging the de-Bruijn graph, MARP demonstrated a scalable, fault-tolerant solution, making it an important reference for designing routing protocols that ensure seamless communication even in the presence of faults. This contributes significantly to improving WSN performance, aligning with broader efforts to enhance network reliability and efficiency in resource-constrained environments.

Birrane et al. (2012) examined the challenges of interplanetary communication networks, where nodes such as orbiters and deep-space relays face mobility constraints, limited resources, and long propagation delays. The study focused on addressing these challenges through the development of Contact Graph Routing (CGR) algorithms. These algorithms leverage deterministic node mobility and pre-configured network information to optimize transmission opportunities. The methodology involved mathematical analysis of the CGR and a multi-destination variant (MD-CGR), revealing that the problem is NP-complete but can be made solvable with certain realistic constraints. The authors highlighted the importance of efficient route creation based on a limited approximation of the contact graph and suggested that brute force approaches, while effective in small networks, cannot scale to larger systems. Findings indicated that CGR-type algorithms, which precompute routes, are crucial for multi-node, multi-path networks in space. However, the need for addressing dynamic changes such as congestion, connection interruptions, and contact graph inaccuracies was also emphasized. The study is relevant to graph-based routing algorithms for enhancing communication efficiency as it demonstrates the applicability of precomputed routing strategies in complex, delay-sensitive networks, highlighting the potential of CGR algorithms to optimize data transmission in interplanetary environments.

Sharma et al. (2013) had investigated opportunistic networks where continuous end-to-end communication paths may not exist. The study aimed to enhance communication efficiency by proposing a contact graph-based forwarding algorithm that considers both delivery performance and resource usage. The methodology involved the aggregation of contact events into weighted graphs to form a contact graph, which helped identify communities within the network. By understanding the contact patterns between nodes, the algorithm could make efficient forwarding decisions based on these community structures. Simulations were conducted using two real-world contact-trace datasets to compare the performance of the proposed approach against existing systems. The findings showed that their method improved delivery success ratios and reduced end-to-end latency while optimizing resource consumption like memory and bandwidth. This research is relevant to the study of graph-based routing algorithms as it demonstrates the application of graph theory in improving communication efficiency through community-based forwarding decisions in dynamic, opportunistic networks.

Kirtiga, R., GnanaPrakasi, O. S., et al. (2014) conducted a study to address the limitations of standard MANET-specific routing protocols in the context of Vehicular Ad Hoc Networks (VANETs). The objective

was to offer a more dependable routing mechanism for VANETs, particularly in highway environments, where the dynamic movement of vehicles leads to frequent link breakages. The methodology involved using a Gaussian distribution model to estimate the connection reliability between vehicular nodes, dynamically updating the network and calculating route reliability values based on these estimates. The study's findings indicated that this approach could reduce link breakages and enhance throughput in environments with fluctuating vehicle velocities. With calculating the most reliable paths for data transmission, the proposed protocol offered a more stable solution than traditional MANET routing protocols. The relevance of this study in the broader context of graph-based routing algorithms lies in its innovative application of probability theory to improve routing reliability in highly dynamic networks like VANETs. It showcases how graph theory can be employed to adapt to the rapid changes in network topology, enhancing communication efficiency by ensuring data is transmitted via the most reliable paths in environments prone to frequent disruptions.

Sakkari (2015) investigated the challenges of coverage and connectivity in wireless sensor networks (WSNs), which have remained unresolved despite significant focus in the research community. The study aimed to enhance communication efficiency by developing a graph-based coverage and connectivity technique (GCCT) that optimized these issues in large-scale WSNs. The methodology included modeling a hierarchical routing protocol that accounted for radio and energy usage, incorporating a novel super-leader node module to enhance energy efficiency. Simulations were conducted to compare GCCT with conventional methods. The findings showed that the GCCT significantly outperformed standard approaches in energy conservation, offering a more efficient solution for WSN communication. This study contributes to the field of graph-based routing by addressing critical challenges in WSNs, particularly the balance between coverage, connectivity, and energy efficiency, which are central to enhancing communication performance in energy-constrained environments.

Liu and Wang (2015) aimed to address the challenge of routing pipes in complex systems like aero-engine engineering by proposing a graph-based routing method. Their methodology involved extending the visibility graph, traditionally used in 2D machine route planning, to the 3D circumferential spaces of aero-engines. They combined geodesics with engineering principles to enhance the algorithm's effectiveness. Two adaptive algorithms were developed to automatically select circumferential layers and areas for pipe routing. The findings revealed that their algorithm identified pipe designs with the shortest overall lengths, outperforming previous methods in both efficiency and computation. Numerical calculations validated the approach, demonstrating that the algorithm reduced pipe lengths and improved the overall design process.In the context of graph-based routing for communication efficiency, this study is relevant as it showcases how graph theory can optimize routing in complex, constrained environments, offering insights for enhancing routing efficiency in network systems.

Kim and Peeta (2016) examined the impact of vehicle-to-vehicle (V2V) communications in improving traffic conditions through a graph-based multilayer network framework. The study's objective was to model a V2V-based Advanced Traveler Information System (ATIS) without central coordination, focusing on how cars update their knowledge of traffic conditions through direct experiences and shared data from other vehicles. The methodology involved constructing three interconnected network layers: physical traffic flow, intervehicle communication, and information flow networks. The intervehicle communication layer was created based on the time-dependent positions of cars, while the information flow network captured how data on traffic conditions propagated among vehicles. Using a graph-based reverse search method, the researchers monitored vehicle knowledge over time, and synthetic experiments compared the graph-based

approach with traditional simulation methods. Their findings indicated that the graph-based method improved memory utilization and reduced processing time, making it an efficient solution for V2V-based traffic management systems. This study is highly relevant to graph-based routing algorithms as it demonstrates how graph structures can be leveraged for real-time, decentralized traffic management, offering a robust framework for enhancing communication efficiency and route optimization in crowded networks through V2V communication techniques.

Hsu, C. F., Chang, Y. C., et al. (2016) conducted a study focused on enhancing communication efficiency in elastic optical networks (AEONs) by addressing the routing and spectrum assignment (RSA) problem, a critical resource management issue akin to the conventional routing and wavelength assignment (RWA) problem. They developed two heuristic algorithms, LG-FF and LG-SP, by modifying the layered graph model, a well-established RWA framework. An analytical model was constructed to estimate the necessary number of layered graphs. The findings indicated that the LG-SP algorithm achieved comparable blocking performance to near-optimal solutions while significantly reducing time complexity. This study demonstrated that the layered graph model serves as an essential representation for resource utilization, effectively addressing both blocking performance and computational efficiency. The research contributes to the field of graph-based routing algorithms by highlighting the importance of innovative heuristic approaches in optimizing resource management in advanced network architectures.

Lu, F., Li, J., et al. (2017), the authors addressed the challenges of effective message transmission in delaytolerant networks (DTNs), particularly in scenarios lacking direct end-to-end connectivity. The methodology involved developing a weighted community graph model to represent the network, transforming it into a community-based structure that reflects the interaction delays between different communities. By applying Dijkstra's algorithm to this community network, the researchers aimed to estimate the minimal transmission latency to target communities. The findings demonstrated the efficacy of the proposed weighted community graph–based social routing method, which incorporates both inter-community and intra-community routing phases. Messages were initially routed to their respective communities based on calculated delays, and then further delivered within communities using social relationships among nodes. Extensive simulations revealed that this approach significantly improved routing performance, particularly in terms of delivery delay and overhead ratio. The relevance of this study lies in its contribution to enhancing communication efficiency in DTNs by leveraging social network structures, thereby providing a robust framework for future research and applications in complex network environments.

Wu et al. (2017) explored the impact of sink mobility on energy efficiency and data collection performance in wireless sensor networks (WSNs). The study aimed to extend network lifespan while ensuring effective data gathering, particularly in hierarchical topologies with random distributions. They developed a data collection method named Double Optimization of Energy Efficiency (DOEE), which utilized a graph-based approach to optimize energy consumption across sensor nodes. Heuristic algorithms were employed to address the optimization challenge presented by DOEE. Additionally, the researchers devised a route discovery protocol and a dynamic gateway protocol to balance energy usage among the sensor nodes. The methodologies were evaluated through simulations using Network Simulation-3. The findings indicated that their proposed algorithm and protocols significantly improved performance, highlighting the effectiveness of graph-based routing strategies in enhancing communication efficiency within WSNs. This study underscores the relevance of optimizing routing algorithms to achieve sustainable and efficient network operations.

Rhim et al. (2018) conducted a study aimed at addressing the challenge of energy consumption in wireless sensor networks (WSNs) by introducing a multi-hop graph-based energy-efficient routing (MH-GEER)

protocol. The researchers developed this protocol to facilitate balanced energy distribution among sensor node clusters, thereby extending network lifespans. They employed a methodology involving centralized cluster construction and distributed selection of cluster heads, building upon principles similar to the Low-Energy Adaptive Clustering Hierarchy (LEACH) approach. The routing phase utilized dynamic multi-hop routing to enhance communication between cluster heads and the base station, making probabilistic decisions based on the energy levels throughout the network. The findings revealed that MH-GEER significantly reduced energy depletion in remote clusters while ensuring load balancing within the network. As a result, the protocol demonstrated improved network lifespan and stability compared to the conventional LEACH protocol, which relied on single-hop routing. This study underscored the relevance of graph-based routing algorithms in enhancing communication efficiency by optimizing energy consumption, making it a significant contribution to the field of WSNs and offering insights into energy-efficient routing protocols for future research.

Lin et al. (2019) explored the dynamics of mobile social networks (MSNs) by proposing a novel priority relation graph (PRG) to evaluate data delivery routing. Their objective was to enhance communication efficiency among mobile users exhibiting social traits, facilitating more effective information distribution. The methodology involved the development of a priority relation graph-based social feature routing (PRG-SFR) method, which identified fragmented multi-paths using the nodes' representation, diameter, and degree. This approach allowed for the establishment of reliable communication paths between senders and receivers, ensuring effective information delivery. The findings indicated that the PRG-SFR algorithm exhibited strong fault tolerance, a favorable forwarding number, reduced transmission duration, and improved delivery rates compared to existing technologies. The numerical analysis demonstrated the algorithm's effectiveness in optimizing data routing within MSNs. The study's relevance lies in its contribution to graph-based routing algorithms, highlighting how social characteristics can enhance communication efficiency. By leveraging the PRG structure, the research underscores the potential for developing robust routing protocols tailored to the unique dynamics of MSNs, ultimately advancing the field of communication networks in complex mobile environments.

III. Graph Representation of a Network with Routing Path

The image below depicts a network graph where nodes (A to F) represent network devices, and edges represent connections with weights indicating the communication cost. The highlighted dashed path from A to F represents the shortest path, optimized by minimizing edge weights, demonstrating efficient routing within the network.



Graph Representation of Network with Shortest Path from A to F

Fig: Network Graph with Shortest Path from Node A to Node F

Here is a network graph illustrating basic graph theory concepts in routing. The nodes (A to F) represent network devices, and edges represent connections with weights indicating the cost or latency between nodes. The highlighted dashed path from A to F represents the shortest path, computed based on the edge weights, showcasing how graph theory can guide efficient routing in a network.

Routing Algorithms in Communication Networks

Routing algorithms are essential in communication networks as they determine the optimal paths for data transmission, ensuring efficient traffic management, minimal delays, and improved network performance.

Dijkstra's Algorithm is commonly used for finding the shortest path in networks with non-negative weights. This algorithm selects the unvisited node with the smallest known distance from the starting point, then calculates the cost to its neighboring nodes, building the shortest path progressively until the destination is reached. It is especially effective in static networks where the topology remains consistent, making it ideal for wired networks and protocols like Open Shortest Path First (OSPF).

The Bellman-Ford Algorithm is useful for networks where negative edge weights may be present. Unlike Dijkstra's, Bellman-Ford allows multiple iterations over each edge, progressively relaxing path costs to find the optimal path. Additionally, it can detect negative cycles, which Dijkstra's algorithm cannot handle. Bellman-Ford is often used in networks with variable link costs, such as wide-area networks (WANs), and is utilized in protocols like the Routing Information Protocol (RIP).

A (A-Star) Algorithm^{*} combines shortest path search with heuristic estimates for more efficient pathfinding. By leveraging a heuristican estimate of the distance to the goalit guides the algorithm toward the destination faster than exhaustive search methods. This makes A^* particularly effective in real-time applications that require quick pathfinding, such as robotics and GPS navigation.

Distance Vector Routing Protocols operate in a decentralized manner, where each node shares its routing table with its neighbors. Every node (or router) periodically updates its table based on information received from neighbors, gradually converging on optimal paths. While suitable for smaller networks due to simplicity, these protocols—such as RIP—are limited in scalability and convergence speed.

Link State Routing Protocols provide routers with a global knowledge of the network topology, enabling them to make more accurate routing decisions. Each router independently calculates the shortest path tree to all nodes based on a complete network map generated from link-state information. Protocols like OSPF use algorithms like Dijkstra to establish paths, which allow for better scalability and faster convergence, making them ideal for large-scale networks.

Ant Colony Optimization (ACO) Algorithms take inspiration from ant foraging behavior to discover efficient paths in dynamic networks. In ACO, artificial agents (or "ants") explore multiple paths, leaving simulated pheromones along their routes. Over time, the optimal paths gather stronger pheromone signals, guiding future "ants" toward these routes. This technique is well-suited for dynamic and decentralized networks, such as mobile ad-hoc networks (MANETs) and IoT systems, due to its flexibility.

Hierarchical Routing divides large networks into clusters or layers, simplifying the routing process and improving scalability. By grouping nodes, with each cluster led by a gateway node, hierarchical routing

reduces the routing table size and processing overhead. This technique is widely applied in large enterprise networks, data centers, and wireless sensor networks to manage routing at scale.

Multipath Routing Algorithms find multiple viable paths for data transmission, distributing data packets across different routes to balance the load and ensure fault tolerance. Instead of a single optimal route, multipath routing enables load distribution, which improves reliability and is particularly useful in high-reliability networks like data centers, MANETs, and streaming services.

Dynamic Source Routing (DSR) is an on-demand algorithm where routes are established only when needed. In DSR, the entire route is embedded in the packet header, allowing the network to adapt quickly to changes in topology. This approach is especially effective in mobile ad-hoc networks (MANETs), where nodes frequently move, creating an ever-changing topology.

Hybrid Routing Protocols combine proactive and reactive strategies, providing both flexibility and efficiency. These protocols proactively maintain routing information for stable paths while discovering new routes reactively as needed. This balanced approach helps manage efficiency and overhead, particularly in networks with variable size and traffic, such as hybrid ad-hoc networks.

Each algorithm is tailored to meet the demands of different network environments, from fixed, predictable networks to dynamic, mobile systems. The choice of routing algorithm depends on factors such as network size, traffic patterns, and stability, balancing speed, scalability, and adaptability for optimal performance.

IV. Graph-Based Routing Algorithms for Enhancing Communication Efficiency

Graph-based routing algorithms play a critical role in enhancing communication efficiency, especially in networks like ad-hoc, wireless sensor networks, and other complex graph-based topologies. Here are key approaches and equations that are often used in graph-based routing to improve communication efficiency

Shortest Path Algorithms

Dijkstra's Algorithm and **Bellman-Ford Algorithm** are classic algorithms that find the shortest path between nodes, optimizing for minimal cost in communication.

The path cost between nodes i and j can be expressed as

$$C(i,j) = \min \sum_{(i,j) \in E} w(i,j)$$

where C(i,j) is the cumulative path cost, w(i,j) is the edge weight, and E represents the set of edges in the graph.

Minimum Spanning Tree (MST) Algorithms

Algorithms like **Kruskal's** or **Prim's** MST are used to reduce the overall communication paths while maintaining connectivity. MST reduces redundant communication channels, which enhances network efficiency by reducing the number of transmissions.

The weight of an MST, W_{MST} , can be formulated as

$$W_{ ext{MST}} = \sum_{(i,j) \in ext{MST}} w(i,j)$$

Energy-Efficient Routing

To reduce the energy consumed during communication, algorithms take into account energy metrics in routing.

For a node i sending data to node j, the **energy consumption** can be modeled as:

$$E(i,j) = d(i,j)^lpha + P_{ ext{idle}}$$

where d(i, j) is the distance, α alpha α is the path-loss exponent (typically $2 \le \alpha \le 4$ in wireless networks), and P_{idle} is the power consumed in the idle state.

Load Balancing

Load balancing across nodes prevents network congestion and improves reliability. Multi-path routing distributes traffic over multiple paths to balance load effectively.

The load balancing function L for a path p can be defined as:

$$L(p) = rac{1}{n}\sum_{i=1}^n rac{f(i)}{c(i)}$$

where f(i) is the flow through node i, c(i) is the capacity of node i, and n is the number of nodes along the path.

Reliability Maximization using Probabilistic Graph Models

Reliability in routing is maximized by modeling the probability of successful transmission along various paths.

For a path with nodes 1, 2...,n, the reliability R is:

$$R=\prod_{i=1}^n r(i,i+1)$$

where r(i,i+1) is the probability of successful transmission between nodes iii and i+1.

V. Conclusion

Graph-based routing algorithms serve as foundational tools in networked communication, offering scalable and efficient solutions across a range of network applications. By utilizing graph theory, these algorithms optimize data transmission, balancing the need for speed, load management, and fault tolerance. From foundational algorithms like Dijkstra's and Bellman-Ford to more complex adaptations for WSNs and VANETs, graph-based routing methods have proven effective in both static and dynamic environments. Studies underscore the adaptability of these algorithms, showing success in applications like energyefficient WSNs, resilient CRNs, and reliable data transfer in challenging environments. As network demands grow, graph-based routing algorithms will continue to be instrumental in driving efficient, faulttolerant, and scalable communication in increasingly complex networked systems.

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