Carle Man Estimate for Coupled Equations

Jyoti, Dr. Poonam Kumari

[1] Research Scholar of Mathematics, [2] Research Supervisor,
Department of Mathematics, Singhania University,
Pacheri Bari, Rajasthan, India.

Published In IJIRMPS (E-ISSN: 2349-7300), Volume 10, Issue 5, (September-October 2022)

Abstract

In this paper, I present an estimate of the Carle Man type for the phase field system with a single observation on the right of the subfield u > gage. Carle men's gauge is not used specifically to show the diffusion coefficient as it shows that the gauge has different weight functions on both the left hand and the right side of the same model with a certain control limit. The creators have also used the multiplier method to identify an inequality like (1.10) with the linear zed phase field system. Nevertheless, we use a rather unique approach which covers various burdens in the main equation for dealing with the additional derived time.

Proper Weight Functions

First, two weight functions are defined that are helpful in monitoring.. First, we assume that an ip = p(x) function is found regularly, with certain characteristics defined on Q.

$$\int_{\Omega}(|f|^2+|\nabla f|^2)dx \le C\int_{Q_\omega}|q_t|^2 dtdx$$
$$+C\int_{\Omega}\left(|\nabla p(x,\theta)|^2+|\Delta p(x,\theta)|^2+|\nabla(\Delta p(x,\theta))|^2\right)dx.$$

$$\psi(x)>0 \;\; \forall x\in\Omega, \quad |\nabla\psi(x)|\ge\zeta>0 \quad \forall x\in\bar{\Omega} \text{ and } \frac{\partial\psi}{\partial\nu}\le 0\;\forall x\in\partial\Omega, \quad (1.11)$$

Whereν denotes the outward normal to $\partial\Omega$. If a function such ip can be established, the weight functions can be introduced now $\phi,\ \alpha : Q\to\mathbb{R}$

$$\phi(x,t)=e^{\lambda\psi(x)}/\beta(t) \text{ and } \alpha(x,t)=\left(e^{2\lambda\|\psi\|_{C(\bar{\Omega})}}-e^{\lambda\psi(x)}\right)/\beta(t), \quad (1.12)$$

Where $\beta(t)=t(T-t)$ Note that weight an is a good weight, with t = 0 and t = T blowing up to +oo. The functions therefore $e^{-2s\alpha}$, $\phi e^{-2s\alpha}$ are smooth and they vanish at = 0 and = and also note that 0 for all $\phi(x,t)\ge C>0$ for all e > 0 and m € R.

$$|\phi_t|\le C(\Omega)T\phi^2, \quad |\alpha_t|\le C(\Omega)T\phi^2 \text{ and } |\alpha_{tt}|\le C(\Omega)T^2\phi^3.$$

We also need the following assessments for the functions to demonstrate the main inequality (p and a: We are now denoting a generic positive constant with C'(fl), Its value varies from line to line and may vary with the ip products and S2, T. You can get the following with simple calculations $|\phi_t| \le C(\Omega)T\phi^2$, $|\alpha_t| \le C(\Omega)T\phi^2$ and $|\alpha_{tt}| \le C(\Omega)T^2\phi^3$. (1.13). Further note that $\nabla\phi = \lambda\phi\nabla\psi$, $\nabla\alpha = -\lambda\phi\nabla\psi$ and $\phi^{-1} \le (T/2)^2$ Allow UM to be an M-bound set throughout this chapter, where M is some positive constant by

$$\mathbb{U}_M = \{k \in L^\infty(\Omega) : \|k\|_{L^\infty(\Omega)} \le M\}. \quad (1.13)$$

Model Translation and Key Estimate

First, the problem (1.14) can be translated in the initial equation into a model without a complicated time derivative qt (note that q is the second equation solution)

$$\left.\begin{array}{ll} p_t - \nabla(a_1(x)\nabla p) + l\nabla(a_2(x)\nabla q) - c_1 p - b_1 q = F, & \text{in } Q, \\ q_t - \nabla(a_2(x)\nabla q) + bq + cp = 0, & \text{in } Q, \\ p(x,\theta) = p_\theta(x),\ q(x,\theta) = q_\theta(x), & \text{in } \Omega, \\ p(x,t) = 0,\ q(x,t) = 0, & \text{on } \Sigma, \end{array}\right\} (1.14)$$

When the functions now apply to the second equation in (1.15) (referred to by (1.14) let q be the solution in (1.15) for general parable balances and assume Assumption 1.15 is true. So for everybody $\lambda \ge \bar{\lambda}_0(\Omega, T) > 0$ there exists a constant $C > 0$ depending on Ω, ω, T, b and c and satisfying

$$\mathcal{I}(q) \le C\Big(\int_Q e^{-2s\alpha}|p|^2 dtdx + s^3\lambda^4 \int_{Q_{\omega_1}} e^{-2s\alpha}\phi^3|q|^2 dtdx\Big), \quad (1.15)$$

Where ω_1 is an open set satisfying $\omega_0 \Subset \omega_1 \Subset \omega$ an

$$\mathcal{I}(q) = (s\lambda)^{-1}\int_Q e^{-2s\alpha}\phi^{-1}(|q_t|^2 + |\Delta q|^2)dtdx + \int_Q e^{-2s\alpha}(s\lambda^2\phi|\nabla q|^2 + s^3\lambda^4\phi^3|q|^2)dtdx.$$

On the other hand, by multiplying (4.2.5) 1 by $t(T-t)$, we get

$$\left.\begin{array}{l} (t(T-t)p)_t - \nabla(a_1(x)\nabla t(T-t)p) - c_1 t(T-t)p \\ \qquad = Ft(T-t) - l\nabla(a_2(x)\nabla q)t(T-t) + b_1 t(T-t)q + p(t(T-t))_t, \\ t(T-t)p = 0 \ \text{ in } \ \Sigma. \end{array}\right\} (1.16)$$

$$t(T - t)p = 0 \ \text{ in } \ \mathrm{E}.$$

Presently, applying the classical Carle man gauge together with the gauge (1.16) for the equation (1.16), we obtain

$$\widetilde{\mathcal{I}}(p) \leq C\Big(\int_Q e^{-2s\alpha}\phi^{-2}|F|^2 dtdx + s^3\lambda^4\int_{Q_{\omega_1}} e^{-2s\alpha}\phi|p|^2 dtdx + \int_Q e^{-2s\alpha}\phi^{-2}|l\nabla(a_2\nabla q) + b_1 q|^2 dtdx\Big), \quad (1.17)$$

for $s \geq CT$ and $\lambda \geq 1$, where

$$\widetilde{\mathcal{I}}(p) = (s\lambda)^{-1}\int_Q e^{-2s\alpha}\phi^{-3}(|p_t|^2+|\Delta p|^2)dtdx+\int_Q e^{-2s\alpha}(s\lambda^2\phi^{-1}|\nabla p|^2+s^3\lambda^4\phi|p|^2)dtdx.$$

Please note that the time factor multiplication changes the weight function forces ϕ in $\mathcal{I}(p)$ New weights in Z result (p). This is now possible to estimate the last term on the right (1.17) as

$$\begin{aligned}\int_Q e^{-2s\alpha}\phi^{-2}|l\nabla(a_2\nabla q) + b_1 q|^2 dtdx &\leq 3\int_Q e^{-2s\alpha}\phi^{-2}(|l\nabla a_2\nabla q|^2 + |la_2\Delta q|^2 + |b_1 q|^2)dtdx \\ &\leq 3s^2\lambda\Big((s\lambda)^{-1}\int_Q e^{-2s\alpha}\phi^{-1}|\Delta q|^2 dtdx + s\lambda^2\int_Q e^{-2s\alpha}\phi|\nabla q|^2 dtdx \\ &\quad + s^3\lambda^4\int_Q e^{-2s\alpha}\phi^3|q|^2 dtdx\Big) \leq s^2\lambda\mathcal{I}(q)\end{aligned}$$

Provided The estimate (1.17) can now be written using the above estimate with (1.16). as

$$\widetilde{\mathcal{I}}(p) \leq C\Big(\int_Q e^{-2s\alpha}\phi^{-2}|F|^2 dtdx + s^3\lambda^4\int_{Q_{\omega_1}} e^{-2s\alpha}\phi|p|^2 dtdx + s^2\lambda\int_Q e^{-2s\alpha}|p|^2 dtdx + s^5\lambda^5\int_{Q_{\omega_1}} e^{-2s\alpha}\phi^3|q|^2 dtdx\Big)$$

Thus for any $s \geq \tilde{s}_0 = \max\{\bar{s}_0, s_1, CT^2\}$ and $\lambda \geq 1$,

$$\widetilde{\mathcal{I}}(p) \leq C\Big(\int_Q e^{-2s\alpha}\phi^{-2}|F|^2 dtdx + s^3\lambda^4\int_{Q_{\omega_1}} e^{-2s\alpha}(\phi|p|^2 + s^2\lambda\phi^3|q|^2)dtdx\Big) \quad (1.18)$$

Now coupling the estimates (1.17) and (1.18), we have

$$\mathcal{I}(q) + \widetilde{\mathcal{I}}(p) \leq C\Big(\int_Q e^{-2s\alpha}\phi^{-2}|F|^2 dtdx + s^3\lambda^4 \int_{Q_{\omega_1}} e^{-2s\alpha}\big(\phi|p|^2 + s^2\lambda\phi^3|q|^2\big)dtdx\Big), \quad (1.19)$$

for the choice of $s \geq \tilde{s}_0$ and $\lambda \geq \tilde{\lambda}_0 = \max\{\bar{\lambda}_0, C\sqrt{T}\}$.

Estimation of $s^3\lambda^4 \int_{Q_{\omega_1}} e^{-2s\alpha}\phi|p|^2 dtdx$

In this section, we estimate the integral $|p|^2$ over Q_{ω_1} on the right-hand side of (1.19) in terms of $|q|^2$ over Q_ω to this end; first we introduce a truncating function $\chi \in C_0^\infty(\Omega)$ satisfying

$$\chi(x) = 1 \text{ in } x \in \omega_1,\ 0 < \chi(x) \leq 1 \text{ in } x \in \omega_2,\ \chi(x) = 0 \text{ in } x \in \Omega\backslash\omega_2, \quad (1.20)$$

Where $\omega_1 \Subset \omega_2 \Subset \omega \Subset \Omega$.

Now we multiply the equation (1.20)

$$s^3\lambda^4 \int_{Q_\omega} ce^{-2s\alpha}\phi\chi|p|^2 dtdx = s^3\lambda^4 \int_{Q_\omega} e^{-2s\alpha}\phi\chi p[-q_t + \nabla(a_2\nabla q) - bq]dtdx$$
$$= I_1 + I_2 + I_3. \quad (1.21)$$

Now we estimate the components T, I = 1, 2, 3 each. Integration in integral T in parts with time, we achieve

$$\begin{aligned} I_1 &= -2s^4\lambda^4 \int_{Q_\omega} e^{-2s\alpha}\phi\chi\alpha_t qp dtdx + s^3\lambda^4 \int_{Q_\omega} e^{-2s\alpha}\phi_t\chi qp dtdx \\ &\quad + s^3\lambda^4 \int_{Q_\omega} e^{-2s\alpha}\phi\chi qp_t dtdx \\ &\leq \delta_1 s^3\lambda^4 \int_{Q_\omega} e^{-2s\alpha}\phi|p|^2 dtdx + \frac{CT^2}{\delta_1}s^5\lambda^4 \int_{Q_\omega} e^{-2s\alpha}\phi^5|q|^2 dtdx \\ &\quad + \frac{CT^2}{\delta_1}s^3\lambda^4 \int_{Q_\omega} e^{-2s\alpha}\phi^3|q|^2 dtdx \\ &\quad + \delta_1(s\lambda)^{-1} \int_{Q_\omega} e^{-2s\alpha}\phi^{-3}|p_t|^2 dtdx + \frac{C}{\delta_1}s^7\lambda^9 \int_{Q_\omega} e^{-2s\alpha}\phi^5|q|^2 dtdx \\ &\leq \delta_1\widetilde{I}(p) + \frac{C}{\delta_1}s^7\lambda^9 \int_{Q_\omega} e^{-2s\alpha}\phi^5|q|^2 dtdx, \end{aligned}$$

Whenever $\lambda \geq 1$ and $s \geq CT(1+\sqrt{T})$ and for $\delta_1 > 0$. In estimating I_2, first we observe that

$$|\nabla(\phi e^{-2s\alpha}\chi)| = |e^{-2s\alpha}(\lambda\phi\nabla\psi\chi + 2s\lambda\phi^2\nabla\psi\chi + \phi\nabla\chi)|$$
$$\leq C(\Omega,\omega)s\lambda e^{-2s\alpha}\phi^2, \text{ for any } s \geq CT^2 \text{ and } \lambda \geq 1.$$

Similarly one can obtain

$$|\nabla(\phi e^{-2s\alpha}\chi)| = |e^{-2s\alpha}(\lambda\phi\nabla\psi\chi + 2s\lambda\phi^2\nabla\psi\chi + \phi\nabla\chi)|$$
$$\leq C(\Omega,\omega)s\lambda e^{-2s\alpha}\phi^2, \text{ for any } s \geq CT^2 \text{ and } \lambda \geq 1.$$

Using the theorem Green and the above estimates, we have

$$\begin{aligned} I_2 \leq\ & \delta_2 s^3\lambda^4 \int_{Q_\omega} e^{-2s\alpha}\phi|p|^2 dtdx + \delta_2 s\lambda^2 \int_{Q_\omega} e^{-2s\alpha}\phi^{-1}|\nabla p|^2 dtdx \\ & + \delta_2(s\lambda)^{-1}\int_{Q_\omega} e^{-2s\alpha}\phi^{-3}|\Delta p|^2 dtdx + \frac{C}{\delta_2}s^7\lambda^8 \int_{Q_\omega} e^{-2s\alpha}\phi^5|a_2|^2|q|^2 dtdx \\ & + \frac{C}{\delta_2}s^7\lambda^9 \int_{Q_\omega} e^{-2s\alpha}\phi^5|a_2|^2|q|^2 dtdx + \frac{C}{\delta_2}s^5\lambda^6 \int_{Q_\omega} e^{-2s\alpha}\phi^3|\nabla a_2|^2|q|^2 dtdx \\ \leq\ & \delta_2\widetilde{I}(p) + \frac{C}{\delta_2}s^7\lambda^9 \int_{Q_\omega} e^{-2s\alpha}\phi^5|q|^2 dtdx \end{aligned}$$

Provided $\lambda \geq C\|a_2\|^2_{L^\infty(\Omega)}$ and $s \geq CT^2\|\nabla a_2\|_{L^\infty(\Omega)}$ and for $\delta_2 > 0$.

estimating the integral I_3, we also get

$$\begin{aligned} I_3 \leq\ & \delta_3 s^3\lambda^4 \int_{Q_\omega} e^{-2s\alpha}\phi|p|^2 dtdx + \frac{C}{\delta_3}s^3\lambda^4 \int_{Q_\omega} e^{-2s\alpha}\phi|b|^2|q|^2 dtdx \\ \leq\ & \delta_3\widetilde{I}(p) + \frac{1}{\delta_3}s^7\lambda^9 \int_{Q_\omega} e^{-2s\alpha}\phi^5|q|^2 dtdx, \end{aligned}$$

for sufficiently large $\lambda \geq 1$ and $s \geq CT^2\|b\|^{1/2}_{L^\infty(Q)}$ and for $\delta_3 > 0$. With the presumption that the c(x) and inequality coefficients are met and 6, = c0/6C for 1 < I < 3 are now read as

$$s^3\lambda^4 \int_{Q_{\omega_1}} e^{-2s\alpha}\phi|p|^2 dtdx \leq C\Big(\int_Q e^{-2s\alpha}\phi^{-2}|F|^2 dtdx + s^7\lambda^9 \int_{Q_\omega} e^{-2s\alpha}\phi^5|q|^2 dtdx\Big), \quad (1.22)$$

For the choice of $\lambda \geq \lambda_0 = \max\{\tilde{\lambda}_0, C\|a_2\|^2_{L^\infty(\Omega)}\}$ and

$s \geq s_0 = \max\{\tilde{s}_0, CT^2(1 + 1/T + \sqrt{T}/T + \|\nabla a_2\|_{L^\infty(\Omega)} + \|b\|^{1/2}_{L^\infty(Q)})\}$.

Eventually, Substituting the inequality (1.22) into (1.23), we obtain the following Carleman estimate.

Theorem 1.2.1 (Carle man Estimate) Let ψ, ϕ *and* α be defined as in (1.22)-(1.23) and the coefficients l,c € UM and b € L°°(Q). Suppose Assumption 4-LI on the coefficients $a_1(x), a_2(x)$ *and* $c(x)$ Just hold true. Hold true. Then exists A0 > 0, so that the following inequality applies to all A > AQ and all s > SQ if a constant C > 0 is p and all p q satisfying p,q € is independent

$$\widetilde{\mathcal{I}}(p) + \mathcal{I}(q) \leq C\Big(\int_Q e^{-2s\alpha}\phi^{-2}|F|^2 dtdx + s^7\lambda^9 \int_{Q_\omega} e^{-2s\alpha}\phi^5|q|^2 dtdx\Big), \quad (1.23)$$

We now have an estimation of the diffuse factor as stability. The inequality of two materials (with the same geometry) estimating the difference between the a \ and a \ and Vtq and VaJ coefficients with the high limit of certain Sobolevqt solution standards and certain p-spatial derivatives at the point = 9, where 9 is a point between 0 and 0. A minimum value point of 1 fp (t). For your convenience, we refer to Q(x,9) as follows: = Q. = Q. In evidence of this stability assessment, the Carle man assessment in the previous section will be the most significant component.

Now set pt = y and qt = z, and you can write the system (1.24) as the solution with the following method (y, z):

$$\left.\begin{array}{ll} y_t + l z_t - \nabla(a_1(x)\nabla y) = F_t, & \text{in } Q, \\ z_t - \nabla(a_2(x)\nabla z) + bz + cy = 0, & \text{in } Q, \\ y(x,\theta) = \widetilde{F},\ z(x,\theta) = \widetilde{G}, & \text{in } \Omega, \\ y(x,t) = 0,\ z(x,t) = 0, & \text{on } \Sigma, \end{array}\right\} \quad (1.24)$$

Where where $F_t = \nabla(f\nabla u_t^*)$, $\widetilde{F} = F_\theta - l z_\theta + \nabla(a_1\nabla p_\theta)$ and $\widetilde{G} = \nabla(a_2(x)\nabla q_\theta) - bq_\theta - cp_\theta$. the proof of the stability gauge follows certain ideas used in the limited domain for the Faltering system and in the unlimited domain for the Schrodinger equation. We will divide the evidence into several straightforward steps in order to clarify the proof of the main outcome by proving them as preliminary results with the following norm.

Assumption 1.3.1 *If ug G C3 (fl) is a true, valuable function, ug and all its derivatives are bound and fulfilled in order of three.* $|\nabla\psi \cdot \nabla u_\theta| \geq C > 0$

Assumption 1.3.2 *Suppose* $|\Delta u^*|, |\nabla(\Delta u^*)|, |\nabla u_t^*|$ *and* $|\Delta u_t^*|$ *are bounded by a positive constant.*

Lemma 1.3.1 Consider the partial differential operator in first order, where ug meets Assumption 1-3.1. Then a constant C > 0, so that the following inequality holds sufficiently large X and s:

$$s^2\lambda^2 \int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-1}|g|^2 dx \leq C \int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-3}|P_0 g|^2 dx,$$

Proof. Following the techniques of let us consider $P_0 g = \nabla u_\theta \cdot \nabla g,$ $e^{-s\alpha_\theta}P_0(e^{s\alpha_\theta}w) = swP_0\alpha_\theta + P_0 w,$ where we note that $g \in H_0^1(\Omega).$

Then, by the formal integration by parts with respective to space variable, we obtain

$$\begin{aligned}
\int_\Omega \phi_\theta^{-3}|Q_0 w|^2 dx &= s^2\int_\Omega \phi_\theta^{-3}|P_0\alpha_\theta|^2|w|^2 dx + \int_\Omega \phi_\theta^{-3}|P_0 w|^2 dx \\
&\quad +2s\int_\Omega \phi_\theta^{-3} w P_0\alpha_\theta P_0 w dx \\
&= s^2\lambda^2\int_\Omega \phi_\theta^{-1}(\nabla u_\theta \cdot \nabla\psi)^2|w|^2 dx + \int_\Omega \phi_\theta^{-3}|P_0 w|^2 dx \\
&\quad -2s\lambda\int_\Omega \phi_\theta^{-2}(\nabla u_\theta \cdot \nabla\psi)(\nabla u_\theta \cdot \nabla w) w dx \\
&\geq s^2\lambda^2\int_\Omega \phi_\theta^{-1}|\nabla u_\theta \cdot \nabla\psi|^2|w|^2 dx - 2s\lambda^2\int_\Omega \phi_\theta^{-2}|P_0\psi|^2|w|^2 dx \\
&\quad +s\lambda\int_\Omega \phi_\theta^{-2}\nabla(P_0\psi\nabla u_\theta)|w|^2 dx.
\end{aligned}$$

Using Assumption 4.3.1, we obtain

$$\int_\Omega \phi_\theta^{-3}|Q_0 w|^2 dx \geq (c_1 s^2\lambda^2 - c_2 T^2 s\lambda - c_3 T^2 s\lambda^2)\int_\Omega \phi_\theta^{-1}|w|^2 dx.$$

Now for any $\lambda \geq 1$ and $s \geq 4(c_2 + c_3)T^2/c_1$, it is clear that

$$s^2\lambda^2 \int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-1}|g|^2 dx \leq C \int_\Omega \phi_\theta^{-3}|Q_0 w|^2 dx = C \int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-3}|P_0 g|^2 dx.$$

The proof of the Lemma 1.3.1 is thus concluded.

Currently, with the assistance of Lemma 4.3.1, we demonstrate the following proposition that provides the key evaluation of the main result.

Proposition 1.3.1 Let (y,z) be the (1-3.1) solution and all the terms of Theorem 4-2.1 and Assumption 1-3.1 must be met. Then a positive constant exists $C(l, \tilde{c}_1)$, such that for sufficiently large s and X, the following estimate holds:

$$s^2\lambda^2 \int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-1}(|f|^2 + |\nabla f|^2)dx \leq C \sum_{i=1}^{4} E_i(\theta),$$

for any $f \in H_0^2(\Omega)$, *where*

$$\begin{aligned} E_1(\theta) &= \int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-3}\left(|y_\theta|^2 + |z_\theta|^2\right)dx, \\ E_2(\theta) &= \int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-3}\left(|\nabla y_\theta|^2 + |\nabla z_\theta|^2\right)dx, \\ E_3(\theta) &= \int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-3}\left(|\nabla p_\theta|^2 + |\Delta p_\theta|^2 + |f\Delta u_\theta^*|^2\right)dx \quad \text{and} \\ E_4(\theta) &= \int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-3}\left(|\nabla p_\theta|^2 + |\Delta p_\theta|^2 + |\nabla(\Delta p_\theta)|^2 + |\nabla(f\nabla u_\theta^*)|^2\right)dx. \end{aligned} \qquad (1.25)$$

Proof. From the value of the solutions of (4.1. 4)! At $t = 9$, we first obtain

$$y_\theta + l z_\theta - \nabla(a_1 \nabla p_\theta) = F_\theta. \qquad (1.25)$$

Now, from Lemma 1.3.1 and recalling $F = \nabla(f(x)\nabla u^*)$, we have the following estimates: first we note that

$$P_0 f = \nabla u_\theta^* \cdot \nabla f = y_\theta + l z_\theta - \nabla(a_1 \nabla p_\theta) - f\Delta u_\theta^*$$

(1.26)

$$s^2\lambda^2 \int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-1}|f|^2 dx \leq C(E_1(\theta) + E_3(\theta)).$$

$$P_0\nabla f = \nabla y_\theta + \nabla(l z_\theta) - \Delta(a_1 \nabla p_\theta) - \nabla(f\Delta u_\theta^*) - \nabla f \Delta u_\theta^*$$

and therefore

$$s^2\lambda^2 \int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-1}|\nabla f|^2 dx \le C(E_2(\theta) + E_4(\theta)), \tag{1.27}$$

Where the constant C depends on l and $\tilde{c}_1$. Further, coupling the estimates (1.27) and (1.28), one can conclude the proof of Proposition 1.28.

Main Results

In the light of Proposal 1.3.1, it is obvious that we need further estimates to reflect the main results. $E_i(\theta)$ for $i = 1, 2, 3, 4$. In these estimates are proved and most of us make use of Carle man's estimate in the last section.

Estimation of $E_1(\theta)$

The evidence of this estimate is that the stability of the illustrative equations is being expanded to include classical ideas. The use of weight (1.27) and inequality of Cauchy and the critical estimation are generated (1.3.7).

Lemma 1.3.2: If all Theorem 1.23 hypotheses are fulfilled. There is then a permanent C > 0, so for everybody $\lambda \ge \lambda_1 > 0$ *and* $s \ge s_1(\Omega, T)$, the following inequality holds:

$$E_1(\theta) \le C\ \mathcal{J}(f, z), \tag{1.28}$$

where

$$\mathcal{J}(f,z) = \int_Q e^{-2s\alpha}\phi^{-2}|F_t|^2 dt dx + s^7\lambda^9 \int_{Q_\omega} e^{-2s\alpha}\phi^5|z|^2 dt dx.$$

Proof. First, noting that $\alpha(0) = +\infty$, we have

$$\begin{aligned}
\int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-3}|z_\theta|^2 dx &= \int_0^\theta \frac{\partial}{\partial t}\Big(\int_\Omega e^{-2s\alpha}\phi^{-3}|z(x,t)|^2 dx\Big) dt \\
&= -2\int_{Q_\theta} s e^{-2s\alpha}\phi^{-3}\alpha_t|z|^2 dt dx + 2\int_{Q_\theta} z z_t e^{-2s\alpha}\phi^{-3} dt dx \\
&\quad -3\int_{Q_\theta} e^{-2s\alpha}\phi^{-4}\phi_t|z|^2 dt dx \\
&\le C(sT^9 + s\lambda T^{16} + T^{11})\int_Q e^{-2s\alpha}\phi^3|z|^2 dt dx + (s\lambda)^{-1}\int_Q e^{-2s\alpha}\phi^{-1}|z_t|^2 dt dx \\
&\le C\mathcal{I}(z), \text{ for any } s \ge C(T^{\frac{9}{2}} + T^8 + T^{\frac{11}{3}}) \text{ and } \lambda \ge 1,
\end{aligned}$$

Where $Q_\theta = \Omega \times (0, \theta)$. Now, applying the estimate (1.26) to the system (1.27),

We get

$$\widetilde{\mathcal{I}}(y) + \mathcal{I}(z) \leq C\ \mathcal{J}(f, z), \quad (1.28)$$

For any s > s0 and for all A > A0, we have

$$\int_{\Omega} e^{-2s\alpha_\theta} \phi_\theta^{-3} |z_\theta|^2 dx \leq C\ \mathcal{J}(f, z). \quad (1.29)$$

Similarly, one can estimate

$$\begin{aligned} \int_{\Omega} e^{-2s\alpha_\theta} \phi_\theta^{-3} |y_\theta|^2 dx &= \int_0^\theta \frac{\partial}{\partial t}\Big(\int_{\Omega} e^{-2s\alpha(x,t)} \phi^{-3} |y(x,t)|^2 dx \Big) dt \\ &\leq C(sT^5 + s\lambda T^8 + T^7) \int_Q e^{-2s\alpha} \phi |y|^2 dt dx + (s\lambda)^{-1} \int_Q e^{-2s\alpha} \phi^{-3} |y_t|^2 dt dx \\ &\leq C\widetilde{\mathcal{I}}(y), \text{ for any } s \geq C(T^{\frac{5}{2}} + T^4 + T^{\frac{7}{3}}) \text{ and } \lambda \geq 1. \end{aligned}$$

Therefore, we also have

$$\int_{\Omega} e^{-2s\alpha_\theta} \phi_\theta^{-3} |y_\theta|^2 dx \leq C\ \mathcal{J}(f, z). \quad (1.30)$$

The evidence for Lemma 1.3.1 can be sufficiently large by coupling estimates (1.30) and (1.31)

$$s \geq s_1 = C(T^{\frac{7}{3}} + T^{\frac{5}{2}} + T^{\frac{11}{3}} + T^4 + T^{\frac{9}{2}} + T^8) \text{ and } \lambda \geq 1.$$

Estimation of $E_2(\theta)$

The estimate is again based on integration by parts with the estimates of the weight function (1.3.1) (1.3.2)

Lemma 4.3.3 *Let $l, c \in \mathbb{U}_M$ and $b \in L^\infty(Q)$ and suppose Assumption 1.1.1 is satisfied. Then there exist $\lambda_2 > 0$ and $s_2 > 0$ constant $C > 0$, such that for all X> X2 and s > s2, the following inequality holds:*

$$E_2(\theta) \leq C s \lambda^2 \mathcal{J}(f, z), \quad (1.31)$$

Where $\mathcal{J}(f,z)$ *is defined in (1-3.6).*

Proof. Let us start the proof by multiplying the equation (1.31) '1 by $\eta(y) :=$

$$e^{-2s\alpha}\phi^{-3}\nabla(a_1\nabla y)$$

And then integrating over *Qs* to obtain

$$\int_{Q_\theta}\eta(y)y_t dtdx = \int_{Q_\theta}\eta(y)(F_t - lz_t + \nabla(a_1\nabla y))dtdx. \tag{1.32}$$

We now estimate separately the left and right sides. First, we integrate parts on the left side of the integral, we get

$$\begin{aligned} -\int_{Q_\theta}\eta(y)y_t dtdx &= \int_{Q_\theta}\nabla(e^{-2s\alpha}\phi^{-3})a_1 y_t\nabla y dtdx + \frac{1}{2}\int_{Q_\theta}e^{-2s\alpha}\phi^{-3}a_1(|\nabla y|^2)_t dtdx \\ &= J_1 + J_2. \end{aligned}$$

But we note that $|\nabla(e^{-2s\alpha}\phi^{-3})| \leq s\lambda e^{-2s\alpha}\phi^{-2}$, for $s \geq CT^2$ and so

$$\begin{aligned} -\int_{Q_\theta}\eta(y)y_t dtdx &= \int_{Q_\theta}\nabla(e^{-2s\alpha}\phi^{-3})a_1 y_t\nabla y dtdx + \frac{1}{2}\int_{Q_\theta}e^{-2s\alpha}\phi^{-3}a_1(|\nabla y|^2)_t dtdx \\ &= J_1 + J_2. \end{aligned}$$

But we note that $|\nabla(e^{-2s\alpha}\phi^{-3})| \leq s\lambda e^{-2s\alpha}\phi^{-2}$, for $s \geq CT^2$ and so

$$\begin{aligned} J_1 &\leq s\lambda\Big(C\|a_1\|^2_{L^\infty(\Omega)}s\lambda\int_{Q_\theta}e^{-2s\alpha}\phi^{-1}|\nabla y|^2 dtdx + (s\lambda)^{-1}\int_{Q_\theta}e^{-2s\alpha}\phi^{-3}|y_t|^2 dtdx\Big) \\ &\leq s\lambda\,\widetilde{I}(y), \text{ for any } \lambda \geq C\|a_1\|^2_{L^\infty(\Omega)} \text{ and } s \geq CT^2. \end{aligned} \tag{1.33}$$

Time integrating by parts in J_2, we get

$$J_2 = -\frac{1}{2}\int_{Q_\theta}(e^{-2s\alpha}\phi^{-3})_t a_1|\nabla y|^2 dtdx + \int_\Omega a_1 e^{-2s\alpha_\theta}\phi_\theta^{-3}|\nabla y_\theta|^2 dx.$$

Here, it is easy to see that $|(e^{-2s\alpha}\phi^{-3})_t| \le Cs\lambda^2 e^{-2s\alpha}\phi^{-1}$ for $\lambda \ge 1$ and $s \ge CT^3$, and therefore

$$J_2 \ge \int_\Omega a_1 e^{-2s\alpha_\theta}\phi_\theta^{-3}|\nabla y_\theta|^2 dx - Cs\lambda^2\int_{Q_\theta} e^{-2s\alpha}\phi^{-1}|\nabla y|^2 dtdx. \quad 1.34)$$

Coming to the right hand side integrals of (1.34), we have

$$\int_{Q_\theta}\eta(y)(F_t - lz_t + \nabla(a_1\nabla y))dtdx = \int_{Q_\theta}\eta(y)F_t dtdx$$
$$+\int_{Q_\theta} e^{-2s\alpha}\phi^{-3}|\nabla(a_1\nabla y)|^2 dtdx - \int_{Q_\theta}\eta(y)lz_t dtdx = J_3 + J_4 + J_5.$$

The above components are then measured one at a time. If the inequality of Cauchy is applied, we get for J_3, J_4:

$$\begin{aligned} J_3 &\le \int_{Q_\theta} e^{-2s\alpha}\phi^{-3}|a_1|^2|\Delta y|^2 dtdx \\ &\quad + CT^4\int_{Q_\theta} e^{-2s\alpha}\phi^{-1}|\nabla a_1|^2|\nabla y|^2 dtdx + CT^2\int_{Q_\theta} e^{-2s\alpha}\phi^{-2}|F_t|^2 dtdx \\ &\le s\lambda^2\left(\widetilde{\mathcal{I}}(y) + \int_{Q_\theta} e^{-2s\alpha}\phi^{-2}|F_t|^2 dtdx\right) \quad (1.35) \end{aligned}$$

$$\begin{aligned} J_4 &\le 2\int_{Q_\theta} e^{-2s\alpha}\phi^{-3}|a_1|^2|\Delta y|^2 dtdx + CT^4\int_{Q_\theta} e^{-2s\alpha}\phi^{-1}|\nabla a_1|^2|\nabla y|^2 dtdx \\ &\le s\lambda^2\widetilde{\mathcal{I}}(y), \quad (1.36) \end{aligned}$$

Whenever $\lambda \ge C(1 + \|a_1\|^2_{L^\infty(\Omega)})$ and $s \ge CT^2(1 + \|\nabla a_1\|_{L^\infty(\Omega)})$. Estimating the integral J5, we also get

$$J_5 \le \int_{Q_\theta} e^{-2s\alpha}\phi^{-3}|a_1|^2|\Delta y|^2 dtdx + CT^4\int_{Q_\theta} e^{-2s\alpha}\phi^{-1}|\nabla a_1|^2|\nabla y|^2 dtdx$$

$$+CT^4 \int_{Q_\theta} e^{-2s\alpha}\phi^{-1}|l|^2|z_t|^2 dtdx \leq s\lambda^2\big(\widetilde{\mathcal{I}}(y)+\mathcal{I}(z)\big), \quad (1.37)$$

For any $s \geq CT^2\|\nabla a_1\|_{L^\infty(\Omega)}$ and $\lambda \geq C(\|a_1\|^2_{L^\infty(\Omega)} + T^4\|l\|^2_{L^\infty(\Omega)})$. easily get the assumption on the a\ coefficient and replace the inequality with an estimation (1.36) in the relation (1.37).

$$\int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-3}|\nabla y_\theta|^2 dx \leq Cs\lambda^2\mathcal{J}(f,z), \quad (1.38)$$

for any $s \geq CT^2(1+T+\|\nabla a_1\|_{L^\infty(\Omega)})$ and $\lambda \geq C(1+\|a_1\|^2_{L^\infty(\Omega)} + T^4\|l\|^2_{L^\infty(\Omega)})$. Next we multiply the equation (1.38)2 bv $\zeta(z) := e^{-2s\alpha}\phi^{-3}\nabla(a_2\nabla z)$ and then integrating over *Qe* to get

$$\int_\Omega e^{-2s\alpha_\theta}\phi_\theta^{-3}|\nabla z_\theta|^2 dx \leq Cs\lambda^2\mathcal{J}(f,z),$$

Further calculations are easy to obtain, as we did with previous calculations and assuming a2.

for any $s \geq C((T^2+T^6)\|\nabla a_2\|_{L^\infty(\Omega)} + T^6\|c\|_{L^\infty(\Omega)} + T^8\|b\|_{L^\infty(Q)})$ and $\lambda \geq C(T^4 + (1+T^8)\|a_2\|^2_{L^\infty(\Omega)})$.

The opposite problem often occurs in many parts of connected mathematics, which require internal or limit measurement to produce the values of some model parameters. This section's main goal is to establish the stability effects for the determination of Two time-independent allegorical compounds for Dirichlet line combined results of phase transitions.

As mathematical models for the transitions of phase, phase field systems have been noted at the latest. In view of several authors (we talk of Penrose and Fife's work and take account of the exhaustive explaination of basic science), we review the phase field models first introduced, and subsequently reexamined and improved from a thermodynamic point of view, to expand the enthalpy technique on Stefan's problem to make it conceivable; In later years, mathematicians have worked incredibly hard to study several versions of the model, with interesting results in the presence and consistencies of solutions, as well as in relying on physical parameters. The easiest linear phase field model can be composed as an explanatory equation system which describes transitions between two states in unadulterated material (u and v) as a solution, for instance strong or fluid.

$$\left.\begin{array}{ll} u_t + l(x)v_t - \Delta u + a(x)v = f_1(x), & \text{in } Q, \\ v_t - \Delta v + b(x,t)v + c(x)u = f_2(x), & \text{in } Q, \\ u(x,\theta)\Big| = u_\theta(x),\ v(x,\theta) = v_\theta(x), & \text{in } \Omega, \\ u(x,t) = h_1(x,t),\ v(x,t) = h_2(x,t), & \text{on } \Sigma, \end{array}\right\} \quad (1.40)$$

For dimension limit 30 of class C2, open-bounded subset of R." The coefficient L € L°(0) for latent heat is b£ Cl(Q); a fixed value of some EUR 9(0,T) is adequately consistent and the semi-initial value ug is sufficiently normal (for example, ug,Vg) £(H2(O))). Non-zero smooth Dirichlet border data Hello: S —> R is kept. and $(f_1, f_2) \in (L^2(\Omega))^2$ are given functions. The response u shows the distribution of the temperature of a surface area 0, which can be strong or fluid in two stage and smooth (when the melting temperature is zero). The phase field function is called V and the purpose is to scal v almost+1 for one phase, for example the fluid phase and v near the -1 for the other high phase. The function of the phase field is clearly visible and differentiates among different phases.

It has also been identified with small amounts in numerous areas of factual mechanics. The request parameters are combined with other variables on a system with complex elements and are constrained to have a fixed value in the weight temperature plane on either end of the balance competition curve. The obscure a(x) and c(x) coefficients are considered smooth enough and are kept free from time t.

In this section, the aim is to achieve the stability estimate of Lipschitz by the internal measurements of one observation in a limited area of dimension n < 3 in de-determining the coefficients a(x) and c(x). An L2-weighted inequality of Carle man type with solutions to phase system solutions will be the key ingredient to these stability results, which is quickly explained later. The phase field system controllability was investigated. We follow the strategy used to obtain a Carle man estimate by means of two observations in various transformations identified with inverse problems and conclude another Carle man estimate by using certain vitality typology.

In a warmth-conducting system, the first temperature and warmth coefficient were simultaneously reconstructed by Yamamoto and Zou Since Carleman's global estimates of the stability of a reverse problem concern the explanatory system, the reverse measurement of a variable coefficient and constant illustrative systems was subsequent combined with one or more reverse measures. Confrontational reconstruction of 1-solution measuring factor over (f0,T) x fi, and some measurements at certain times, T'€ and initial reaction-diffusion system conditions will take place, for example (t0, T). Conversely, all (or some) coefficients of the reaction diffusion convection system were examined by observations of arbitrary sub-specific observations over a time interval of only one component and two components at fixed positive time 0.

References

[1] V. Barbu, Local controllability of the phase field system, Nonlinear Analysis: Theory, Methods & Applications, 50 (2002), 363-372.

[2] A. Belmiloudi, Parameter identification problems and analysis of the impact of porous media in biofluid heat transfer in biological tissues during thermal therapy, Nonlinear Analysis: Real World Applications, 11 (2010), 1345-1363.

[3] A. Benabdallah, M. Cristofol, P. Gaitan & M. Yamamoto, Inverse problem for a parabolic system with two components by measurements of one component, Applicable Analysis, 88 (2008), 683-709.

[4] N.F. Britton, Reaction Diffusion Equations and their Applications to Biology, Academic Press, London, 1986.

[5] A.L. Bukhgeim & M.V. Klibanov, Uniqueness in the large class of multi-dimensional inverse problems, Soviet Mathematics-Doklady, 24 (1981), 244-247.

[6] I. Bushuyev, Global uniqueness for inverse parabolic problems with final observation, Inverse Problems, 11 (1995), L11.

[7] G. Caginalp, An analysis of a phase field model of a free boundary, Archive for Rational Mechanics and Analysis, 92 (1986), 205-245.