Graphical Overview of Non-linear Diophantine Equation

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Abstract
In this paper we can overview some beautiful Non-Linear Diophantine Equations with Graphical approach and we have tried to deeply study about the Diophantus's Non-Linear Diophantine Equation and its characteristic with some graphical knowledge.

Keywords: - Desmos Graphing Calculator, Co-ordinate axis, Factor method.

Mathematical Subject Classification: - 97F10, 97F20, 97F60.

Published in IJIRMPS (E-ISSN: 2349-7300), Volume 11, Issue 4, July-August 2023

Introduction: A Diophantine equation is one that only accepts integer solutions.
The tenth problem of Hilbert addressed whether there was an algorithm for determining whether any given Diophantine equation has a solution. First-order Diophantine equations can be solved using such a technique. However, Yuri Matiyasevich demonstrated in 1970 that the connection n=F_(2m) (where F_(2m) is the $(2m)^{th}$ Fibonacci number) is Diophantine, demonstrating the difficulty of finding a general solution (Matiyasevich 1970, Davis 1973, Davis and Hersh 1973, Davis 1982, Matiyasevich 1993). More specifically, Matiyasevich demonstrated that if there are any numbers x, y, z, such that P (n, m, x, y, z,...) =0, then the polynomial P in n, m, and a number of other variables x, y, z,... has the condition that n=F_(2m). Matiyasevich particularly demonstrated that if there are any integers x, y, z,... such that P(n,m,x,y,z,...)=0, then there exists a polynomial P in n, m, and a number of other variables, x, y, z,...
The output of Matiyasevich bridged a significant gap in earlier work by Martin Davis, Hilary Putnam, and Julia Robinson. Matiyasevich and Robinson's further research established that there is no method that can determine whether there is a solution, even for equations with thirteen variables. After that, Matiyasevich enhanced this finding to equations with just nine variables (Jones and Matiyasevich 1982). Numerous known and unknowable solutions Diophantine equations are provided by Ogilvy and Anderson in 1988.
An equation of the general form is a linear Diophantine equation (in two variables).
A, B, and C integer solutions are sought for the equation $ax + by = c$. Such equations are perfectly solvable, and Brahmagupta created the first known solution to one of these equations.

- In this paper we have tried to study sum graphical demonstration of Non-Linear Diophantine Equation.

Examples of some Non-Linear Diophantine Equation: -

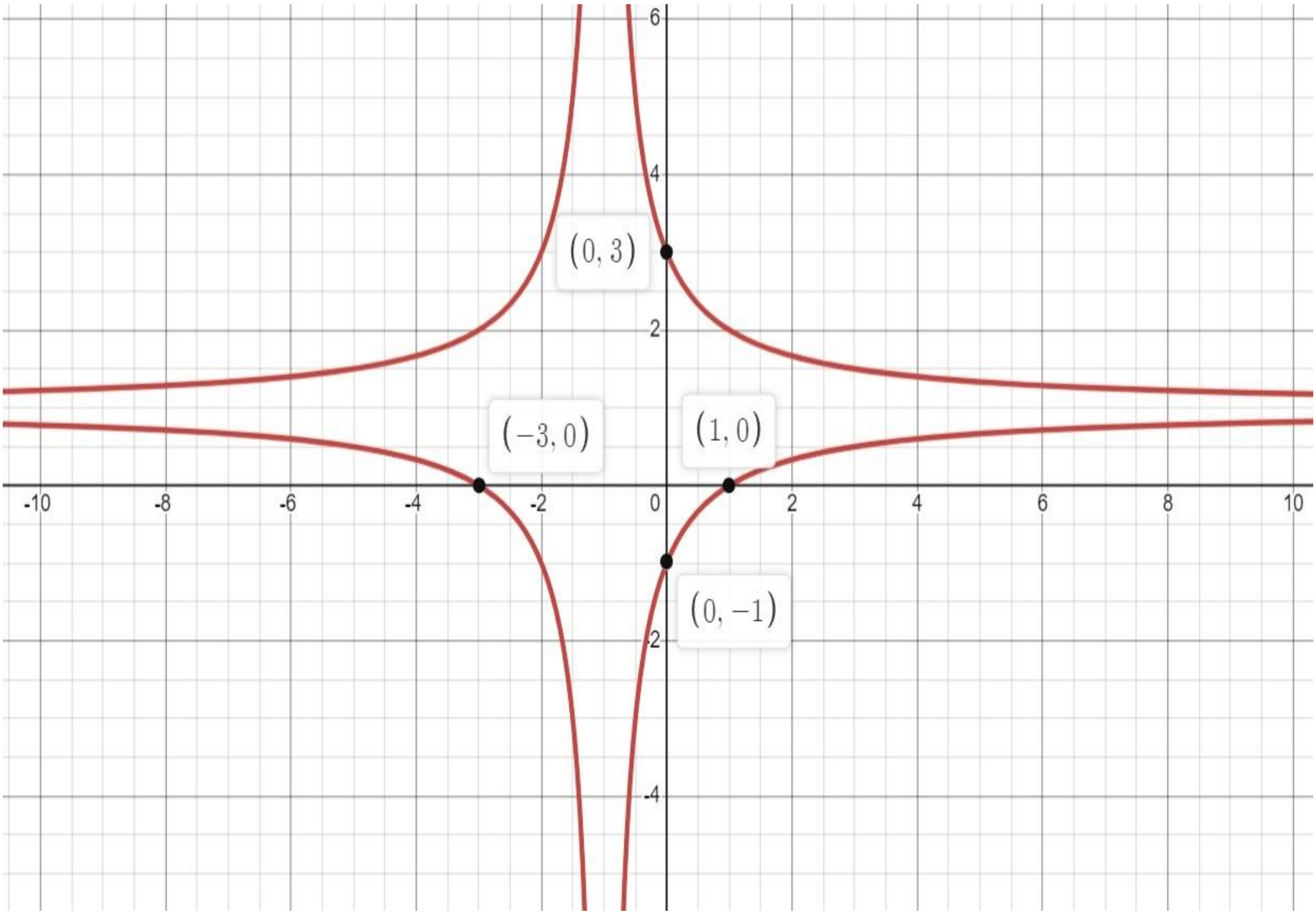
(1) $(x^2+1)(y^2+1)+2(x-y)(1-xy)=4(1+xy)$.
(2) $(xy-7)^2=x^2+y^2$
(3) $x^2(y-1)+y^2(x-1)=1$

etc………...

Graphical approach to solve a Non-Linear Diophantine Equation: -

Theorem: - Any Diophantine equation of unknown variables (x, y) if we can draw the graph of the equation and if it intersects the co-ordinate axes into one or more points than these points where the graph intersects will be the **at least** one solution of the equation.

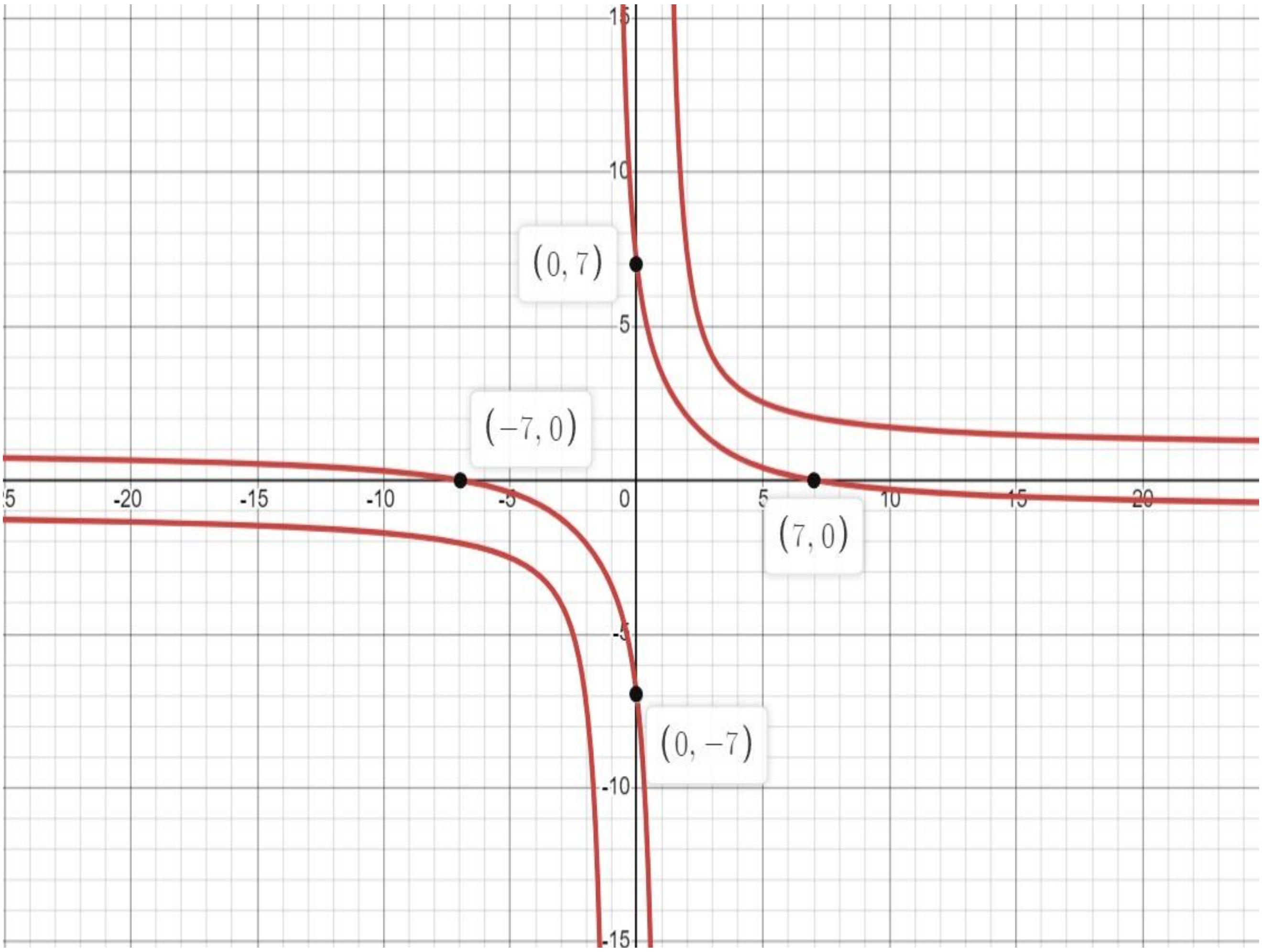
Ex: - (1) $(x^2+1)(y^2+1)+2(x-y)(1-xy)=4(1+xy)$.



Figer-01 by Using Desmos Graphing Calculator

So, we can see here $(0,3), (-3,0), (1,0), (0,-1)$ all are solutions.
But when we solve this problem by factor method the solution is as below: -
$(0,-1), (-2,-1), (1,0), (1,2), (-3,0), (0,3), (-2,3), (-3,2)$.
So, we see four pairs of solution are missing therefore in this approach we can't get all the Diophantine solution.

(2) $(xy-7)^2 = x^2 + y^2$.



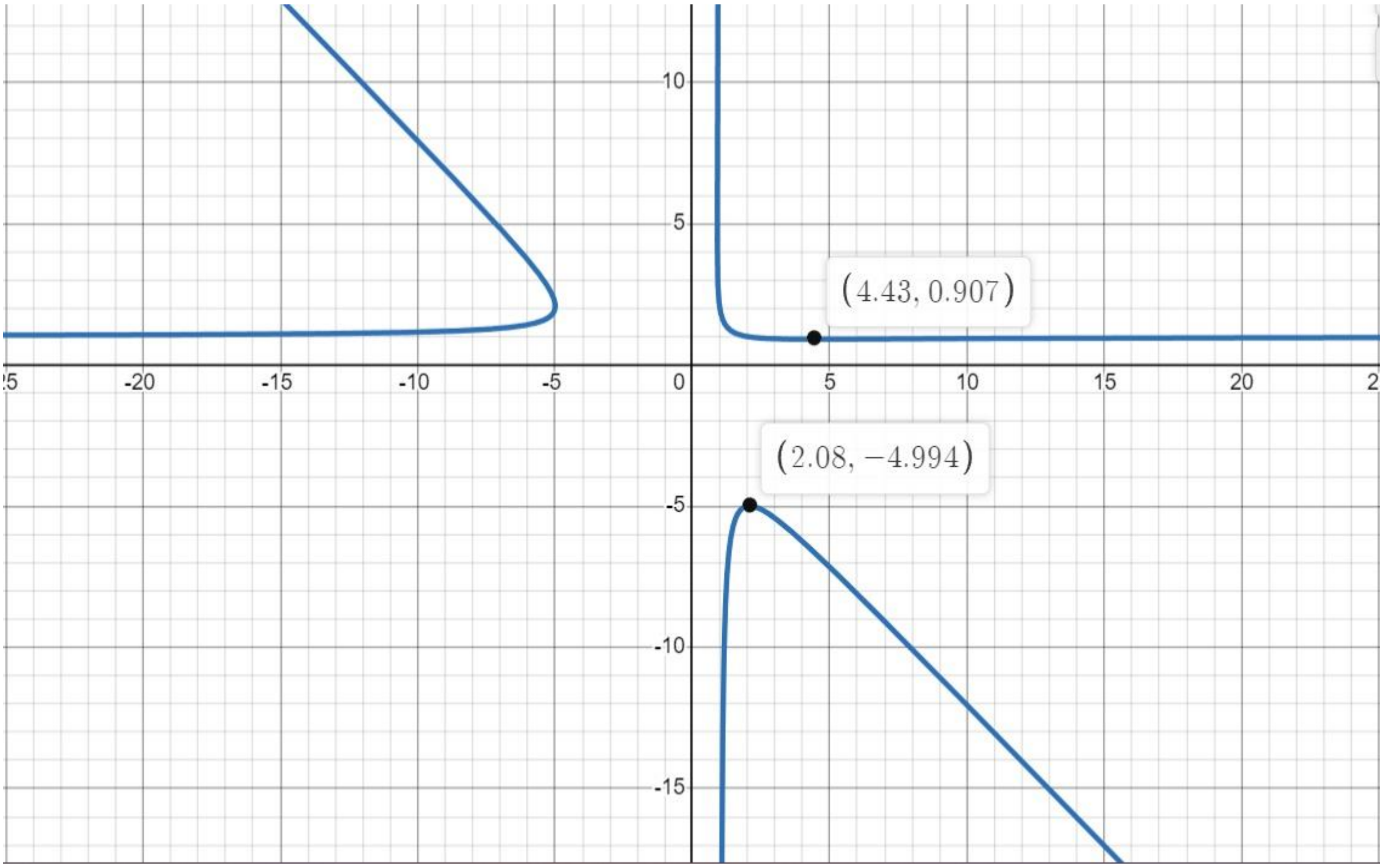
Figer-02 by Using Desmos Graphing Calculator

So, we can see here $(0,7), (7,0), (-7,0), (0,-7)$ all are solutions.
But when we solve this problem by factor method the solution is as below: -

$(1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (7,0), (0,7), (-7,0), (0,-7)$

So, we see six pairs of solution are missing therefore in this approach we can't get all the Diophantine solution.

(3) $x^2(y-1)+y^2(x-1)=1.$



Figer-03 by Using Desmos Graphing Calculator

Here, the graph does not intersect any of the point on the co-ordinate axes. So, we can't conclude anything about the solution of this problem.
But in factor method we can get
$(1,2),(2,1),(-5,2),(2,-5)$, be the solution of this problem.

Advantages: -

(1) If we know the Graph, we can easily conclude the solution of the Diophantine Equation.

(2) No calculation needed.

Disadvantages: -

(1) We can't draw all the graph by hand.

(2) In question number 3 we can't conclude anything about the solution.

(3) All solution is not necessarily given by this approach.

Conclusion: In this method without any lengthy, complicated calculation we can determine a pretty good solution. So, anyone can easily assimilation the method.

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